

## Problem of the Week

### Problem D and Solution

#### Dividing a Square

#### Problem

The large square  $ABCD$  (shown above) is divided into three regions, a smaller square  $BFGH$  and two rectangles  $AHGE$  and  $CDEF$ . The area of square  $BFGH$  is  $36 \text{ cm}^2$ .

We are also given that:

$$\text{Area of } BFGH - \text{Area of } CDEF = \text{Area of } CDEF - \text{Area of } AHGE$$

Determine the areas of rectangle  $CDEF$  and rectangle  $AHGE$ .

#### Solution

##### Solution 1

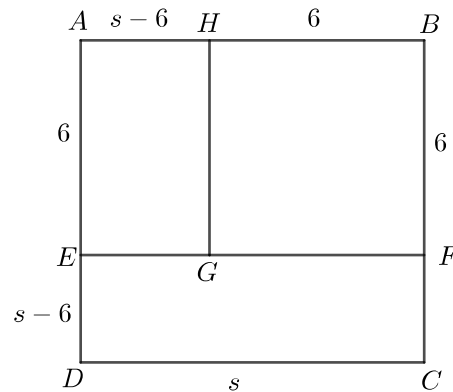
We will use the diagram to the right in our solution.

Let the side length of  $ABCD$  be  $s$  cm.

Since the area of  $BFGH = 36 \text{ cm}^2$ , then  $HB = BF = 6$  cm.

Therefore,  $AE = BF = 6$  cm and  $AH = AB - HB = (s - 6)$  cm.

Similarly,  $ED = (s - 6)$  cm.



Using the given equation,

$$\begin{aligned} \text{Area of } BFGH - \text{Area of } CDEF &= \text{Area of } CDEF - \text{Area of } AHGE \\ 36 - s(s - 6) &= s(s - 6) - 6(s - 6) \\ 36 - s^2 + 6s &= s^2 - 6s - 6s + 36 \\ 18s &= 2s^2 \\ 9 &= s \text{ (since } s > 0) \end{aligned}$$

Therefore, the area of  $CDEF = s(s - 6) = 9(9 - 6) = 27 \text{ cm}^2$ , and the area of  $AHGE = 6(s - 6) = 6(9 - 6) = 18 \text{ cm}^2$ .

**NOTE:** A second solution that uses a quadratic equation is shown on the next page.





## Solution 2

We will use the diagram to the right.

Let the side length of  $AH$  be  $x$ .

Since the area of  $BFGH = 36 \text{ cm}^2$ , then  $HB = BF = 6 \text{ cm}$ .

Therefore  $AE = BF = 6 \text{ cm}$  and  $CD = AH + HB = (x + 6) \text{ cm}$ .

Since  $ABCD$  is a square,  $AD = CD = (x + 6) \text{ cm}$  and  $ED = AD - AE = (x + 6) - 6 = x \text{ cm}$ .

Using the equation:

Area of  $BFGH - \text{Area of } CDEF = \text{Area of } CDEF - \text{Area of } AHGE$ , we get

$$36 - x(x + 6) = x(x + 6) - 6x$$

$$36 - x^2 - 6x = x^2 + 6x - 6x$$

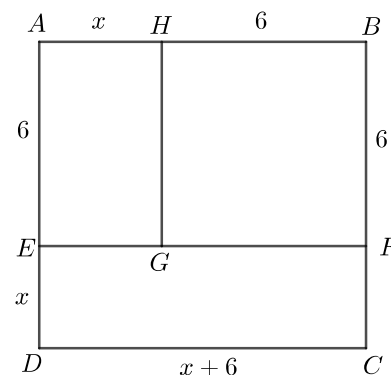
$$2x^2 + 6x - 36 = 0$$

$$2(x^2 + 3x - 18) = 0$$

$$2(x + 6)(x - 3) = 0$$

$$x = -6 \text{ or } 3$$

$$x = 3 \text{ (since } x > 0)$$



Therefore the area of  $CDEF = x(x + 6) = 3(3 + 6) = 27 \text{ cm}^2$  and area of  $AHGE = 6x = 6(3) = 18 \text{ cm}^2$ .

