# Problem of the Week

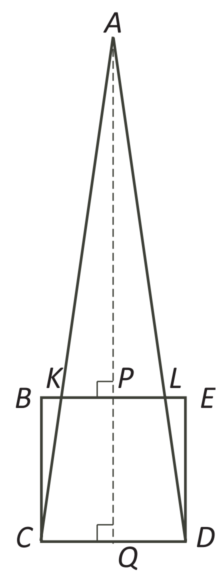
## Problem D and Solution

## Three Polygons

### Problem

In the diagram below, the area of $\triangle ACD$ is twice the area of square $BCDE$. $AC$ and $AD$ meet $BE$ at $K$ and $L$ respectively.

If the side length of the square is 12 cm, determine the area of trapezoid $KCDL$.




### Solution

To find the area of a trapezoid, multiply the sum of the lengths of the two parallel sides, $KL$ and $CD$, by the height, $BC$, and divide the product by 2. To solve this problem we need to find the length of $KL$. Let $x$ represent the length of $KL$.

Draw $APQ$ perpendicular to $KL$ and $CD$. It follows that $AP$ is an altitude of $\triangle AKL$ and $AQ$ is an altitude of $\triangle ACD$.

$$\begin{aligned}
\text{Area of square } BCDE &= 12 \times 12 = 144 \text{ cm}^2 \\
\text{Area } \triangle ACD &= 2 \times \text{Area of Square } BCDE = 288 \text{ cm}^2 \\
\text{But Area } \triangle ACD &= CD \times AQ \div 2 \\
\therefore 288 &= 12 \times AQ \div 2 \\
288 &= 6(AQ) \\
AQ &= 48 \text{ cm}
\end{aligned}$$

Since $AQ = 48$ and $PQ = BC = 12$, then $AP = AQ - PQ = 48 - 12 = 36$ cm.

$$\begin{aligned}
\text{Area of trapezoid } KCDL + \text{Area of } \triangle AKL &= \text{Area } \triangle ACD \\
(KL + CD) \times BC \div 2 + KL \times AP \div 2 &= 288 \\
(x + 12)(12) \div 2 + x(36) \div 2 &= 288 \\
6(x + 12) + 18x &= 288 \\
6x + 72 + 18x &= 288 \\
24x &= 216 \\
x &= 9 \text{ cm}
\end{aligned}$$

$$\begin{aligned}
\text{Area of trapezoid } KCDL &= \frac{(KL + CD) \times PQ}{2} \\
&= \frac{(9 + 12)(12)}{2} \\
&= 126 \text{ cm}^2
\end{aligned}$$

Therefore the area of trapezoid $KCDL$ is $126 \text{ cm}^2$.

**Notes:**

1. In order to find the length of $KL$, we could establish that $\triangle ACD \sim \triangle AKL$. From this we can use the fact that the ratio of the altitudes of the two triangles equals the ratio of the corresponding sides in the two similar triangles. The reader may wish to justify this "fact".

$$\begin{aligned} \frac{AP}{AQ} &= \frac{KL}{CD} \\ \frac{36}{48} &= \frac{x}{12} \\ \frac{3}{4} &= \frac{x}{12} \\ \therefore x &= 9 \text{ cm} \end{aligned}$$

2. Instead of using the formula to determine the area of the trapezoid, we could find the area by subtracting the area of $\triangle AKL$ from the area of $\triangle ACD$.

$$\begin{aligned} \text{Area of trapezoid } KCDL &= \text{Area } \triangle ACD - \text{Area of } \triangle AKL \\ &= 288 - \frac{(KL)(AP)}{2} \\ &= 288 - \frac{9 \times 36}{2} \\ &= 288 - 162 \\ &= 126 \text{ cm}^2 \end{aligned}$$