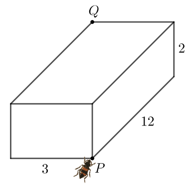


Problem of the Week

Problem D and Solution

Block Walk

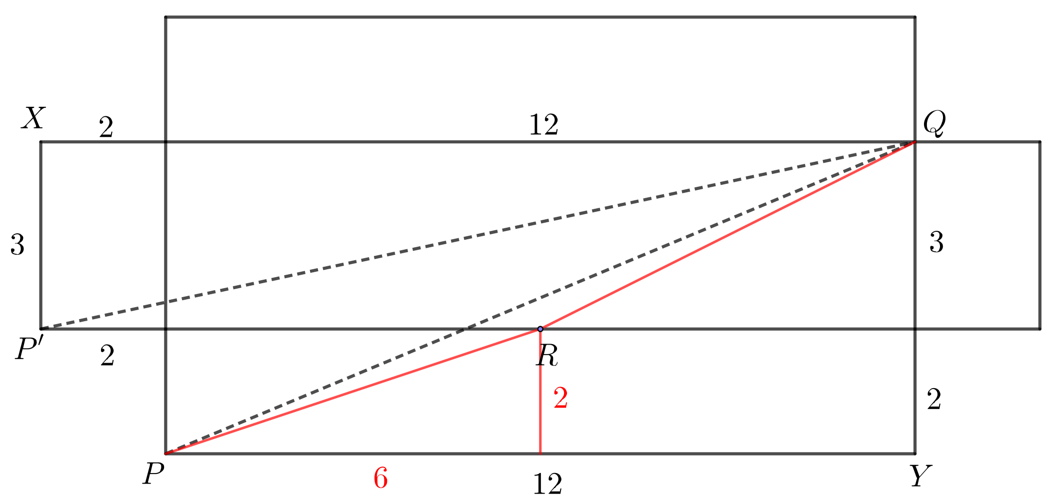
Problem

A beetle walks on the surface of the $2 \times 3 \times 12$ rectangular prism shown. The beetle wishes to travel from P to Q. What is the length of the shortest path from P to Q that the beetle could take?

Solution

Many strategies could be attempted. Perhaps the beetle walks along the edges and travels $12 + 2 + 3 = 15$ units. Perhaps the beetle travels across the right side of the prism from P to the midpoint of the top edge (marked R on the diagram below) and then across the top of the prism to Q. Referring to the diagram below, it can be shown that this distance is $PR + RQ = \sqrt{40} + \sqrt{45} \doteq 13.03$ units. But is this the shortest distance?

To visualize the possible routes, fold out the sides of the box so that they are laying on the same plane as the top of the box. Label the diagram as shown below. Note that as a result of folding out the sides, corner P appears twice. The second corner is labelled P'.



The shortest distance for the beetle to travel is a straight line from P to Q or P' to Q. So both cases must be considered.

PQ is the hypotenuse of right-angled triangle PYQ. Using Pythagoras' Theorem,

$$PQ^2 = PY^2 + YQ^2 = 12^2 + 5^2 = 169 \text{ and } PQ = 13 \text{ follows.}$$

$P'Q$ is the hypotenuse of right-angled triangle $P'XQ$. Using Pythagoras' Theorem,

$$(P'Q)^2 = (P'X)^2 + XQ^2 = 3^2 + 14^2 = 205 \text{ and } P'Q = \sqrt{205} \doteq 14.31 \text{ follows.}$$

Since $PQ < P'Q$, the shortest distance for the beetle to travel is 13 units on the surface of the block in a straight line from P to Q.

This problem is quite straight forward once the three-dimensional nature of the problem is removed.