Problem of the Week

Problem E and Solution

A Tale of Two Cities

Problem

Two cities, Mytown and Yourtown, had the same population at the end 2015. The population of Mytown decreased by 2.5% from the end of 2015 to the end of 2016. Then, the population increased by 8.4% from the end of 2016 to the end of 2017. The population of Yourtown increased by $r\%, r > 0$, from the end of 2015 to the end of 2016. Then, the population of Yourtown increased by $(r + 2)\%$ from the end of 2016 to the end of 2017. Surprisingly, the populations of both cities were the same again at the end of 2017. Determine the value of r correct to one decimal place.

Solution

Let p be the population of Mytown at the end of 2015. Since Mytown and Yourtown have the same population size, then p is also the population of Yourtown at the end of 2015.

The population of Mytown decreased by 2.5% in 2016, so the population at the end of 2016 is $p - \frac{2.5}{100}p = \left(1 - \frac{2.5}{100}\right)p = 0.975p$.

The population of Mytown then increased by 8.4% during 2017, so the population at the end of 2017 is $0.975p + \left(\frac{8.4}{100}\right)(0.975p) = \left(1 + \frac{8.4}{100}\right)(0.975p) = 1.084(0.975p) = 1.0569p$.

The population of Yourtown increased by $r\%$ in 2016, so the population at the end of 2016 is $p + \frac{r}{100}p = \left(1 + \frac{r}{100}\right)p$.

The population of Yourtown then increased by $(r + 2)\%$ during 2017, so the population at the end of 2017 is $\left(1 + \frac{r}{100}\right)p + \frac{r+2}{100}\left(1 + \frac{r}{100}\right)p = \left(1 + \frac{r}{100}\right)p\left(1 + \frac{r+2}{100}\right)$.

Since the populations of Mytown and Yourtown are equal at the end of 2017, we have

$$\begin{aligned}
\left(1 + \frac{r}{100}\right)\left(1 + \frac{r+2}{100}\right)p &= 1.0569p \\
\left(\frac{100 + r}{100}\right)\left(\frac{100 + r + 2}{100}\right) &= 1.0569, \quad \text{dividing both sides by } p, \text{ since } p > 0 \\
(100 + r)(102 + r) &= 10\,569, \quad \text{multiplying both sides by } 10\,000 \text{ to clear fractions} \\
10\,200 + 202r + r^2 &= 10\,569 \\
r^2 + 202r - 369 &= 0
\end{aligned}$$

After using the quadratic formula and ruling out an inadmissible r value, we obtain $r = 1.8\%$, correct to one decimal place.