

 (a, b, c)

Problem of the Week

Problem E and Solution

Not As Easy As a, b, c

Problem

Three real numbers, a, b and c , have a sum of 158 and a product of 74 088. Also, $b = ar$ and $c = ar^2$, for some real values of r . Find all ordered triples (a, b, c) .

Solution

Since $b = ar$, $c = ar^2$ and $abc = 74\,088$, then $a(ar)(ar^2) = a^3r^3 = 74\,088$, or $(ar)^3 = 74\,088$, or $ar = 42$.

Therefore, since $b = ar$, we have $b = 42$.

Now, $a + b + c = 158$ becomes $a + 42 + c = 158$, or $a + c = 116$.

Since $b = ar$, then $42 = ar$, or $r = \frac{42}{a}$ (since the product of a, b and c is not zero, we know $a \neq 0$).

Therefore, $c = a \left(\frac{42}{a}\right)^2 = a \left(\frac{1764}{a^2}\right) = \frac{1764}{a}$.

Substituting $c = \frac{1764}{a}$ into $a + c = 116$, we have

$$a + \frac{1764}{a} = 116$$

$$a^2 + 1764 = 116a$$

$$a^2 - 116a + 1764 = 0$$

$$(a - 18)(a - 98) = 0$$

Therefore, $a = 18$ or $a = 98$.

When $a = 18$, then $r = \frac{42}{18} = \frac{7}{3}$, and one ordered triple is $(18, 42, 98)$.

Indeed, we can check that $18 + 42 + 98 = 158$ and $(18)(42)(98) = 74\,088$.

When $a = 98$, then $r = \frac{36}{98} = \frac{3}{7}$, and one ordered triple is $(98, 42, 18)$.

Indeed, we can check that $98 + 42 + 18 = 158$ and $(98)(42)(18) = 74\,088$.

In conclusion there are two ordered triples that satisfy the conditions of the problem: $(18, 42, 98)$ and $(98, 42, 18)$.

