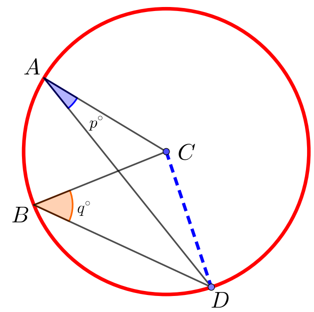
Problem of the Week

Problem E and Solution

Can You Relate?



Problem

Points A, B and D lie on the circumference of a circle with centre C. $\angle CAD = p^\circ$ and $\angle CBD = q^\circ$. Determine the measure of $\angle ACB$ and the measure of $\angle ADB$. What is the relationship between $\angle ACB$ and $\angle ADB$?

Solution

We start by constructing radius CD.

CA and CD are both radii of the circle, so $CA = CD$. Then $\triangle CAD$ is isosceles and $\angle CDA = \angle CAD = p^\circ$. Since the angles in a triangle add to 180°, $\angle ACD = (180 - 2p)^\circ$.

CB and CD are both radii of the circle, so $CB = CD$. Then $\triangle CBD$ is isosceles and $\angle CDB = \angle CBD = q^\circ$. Since the angles in a triangle add to 180°, $\angle BCD = (180 - 2q)^\circ$.

We will now find the measure of $\angle ADB$ and of $\angle ACB$ in order to determine the relationship.

$$\begin{aligned} \angle ADB &= \angle CDB - \angle CDA \\ &= (q - p)^\circ \\ \angle ACB &= \angle ACD - \angle BCD \\ &= (180 - 2p)^\circ - (180 - 2q)^\circ \\ &= (2q - 2p)^\circ \\ &= 2 \times (q - p)^\circ \\ &= 2 \times \angle ADB \end{aligned}$$

$\therefore$ $\angle ACB$ is double the size of $\angle ADB$.

In general, the angle inscribed at the centre of a circle is twice the size of the angle inscribed at the circumference by the same chord. In the following diagram, $\angle ACB$ is inscribed at the centre of the circle by chord AB and $\angle ADB$ is inscribed at the circumference by the same chord. Therefore, $\angle ACB = 2\angle ADB$.