



## Problem of the Week

### Problem E and Solution

#### Acres A Way

#### Problem

Angela and Barry share a piece of land. The ratio of the area of Angela's portion of land to the area of Barry's portion of land is  $3 : 2$ . They each grow corn and peas on their piece of land. The entire piece of land is covered by corn and peas in the ratio  $7 : 3$ . On Angela's portion of the land, the ratio of corn to peas is  $4 : 1$ . What is the ratio of corn to peas for Barry's portion of land?

#### Solution

##### Solution 1

Suppose that Angela and Barry share 100 hectares of land. (We may assume any convenient total area.)

Since the ratio of the area of Angela's land to the area of Barry's land is  $3 : 2$ , then Angela has  $\frac{3}{5}$  of the 100 hectares, or 60 hectares. Barry has the remaining 40 hectares.

Since the entire piece of land is covered by corn and peas in the ratio  $7 : 3$ , then  $\frac{7}{10}$  of the 100 hectares (that is, 70 hectares) is covered with corn and the remaining 30 hectares with peas.

On Angela's land, the ratio of corn to peas is  $4 : 1$ , so  $\frac{4}{5}$  of her 60 hectares, or 48 hectares, is covered with corn and the remaining 12 hectares with peas.

Since there are 70 hectares of corn in total, then Barry has  $70 - 48 = 22$  hectares of corn.

Since there are 30 hectares of peas in total, then Barry has  $30 - 12 = 18$  hectares of peas.

Therefore, the ratio of corn to peas on Barry's land is  $22 : 18 = 11 : 9$ .

##### Solution 2

Suppose that the total combined area of land is  $x$ .

Since the ratio of the area of Angela's land to the area of Barry's land is  $3 : 2$ , then Angela has  $\frac{3}{5}$  of the land, or  $\frac{3}{5}x$ , while Barry has the remaining  $\frac{2}{5}x$ .

Since the entire piece of land is covered by corn and peas in the ratio  $7 : 3$ , then  $\frac{7}{10}x$  is covered with corn and the remaining  $\frac{3}{10}x$  with peas.

On Angela's land, the ratio of corn to peas is  $4 : 1$  so  $\frac{4}{5}$  of her  $\frac{3}{5}x$ , or  $\frac{4}{5} \left( \frac{3}{5}x \right) = \frac{12}{25}x$ , is covered with corn and the remaining  $\frac{3}{5}x - \frac{12}{25}x = \frac{3}{25}x$  with peas.

Since the area of corn is  $\frac{7}{10}x$  in total, then Barry's area of corn is  $\frac{7}{10}x - \frac{12}{25}x = \frac{11}{50}x$ .

Since the area of peas is  $\frac{3}{10}x$  in total, then Barry's area of peas is  $\frac{3}{10}x - \frac{3}{25}x = \frac{9}{50}x$ .

Therefore, the ratio of corn to peas on Barry's land is  $\frac{11}{50}x : \frac{9}{50}x = 11 : 9$ .

