# Problem of the Week
Problem E and Solution
Going Bananas

Problem

A maple tree is surrounded by two pine trees and two palm trees, as shown above. Five types of bananas, P, Q, R, S, T are placed in the trees. Each tree has exactly one type of banana in it and there is a different type of banana in each tree. A monkey is hungry and takes the same amount of time to eat any banana. The monkey starts on a tree and begins by eating a banana on that tree. The monkey then swings to another tree and eats a banana on that tree. The monkey then swings to another tree and eats a banana on that tree. This will continue until the monkey stops. It takes the monkey three seconds to swing from the maple tree to any other tree or vice versa, two seconds to swing from a pine tree to a palm tree or vice versa, and seven seconds to swing between two pine trees or two palm trees while avoiding the maple tree along the way. The monkey eats bananas of type P, Q, S, R, T, R, P in that order then stops. List all the possible types of banana can be in the maple tree if the total amount of time the monkey swings is as small as possible.

Solution

The given sequence P, Q, S, R, T, R, P visits all the banana types with six swings $P - Q$, $Q - S$, $S - R$, $R - T$, $T - R$, $R - P$. For this sequence, the monkey swings to each tree at least once and from each tree at least once. Therefore, there needs to be a swing to and a swing from the maple. So the minimum time that is possible occurs when there are only two swings to and from the maple and the other four swings are between the pine and palm trees. Thus, the minimum possible time is $2(3) + 4(2) = 14$ seconds. We can show three such routes below.
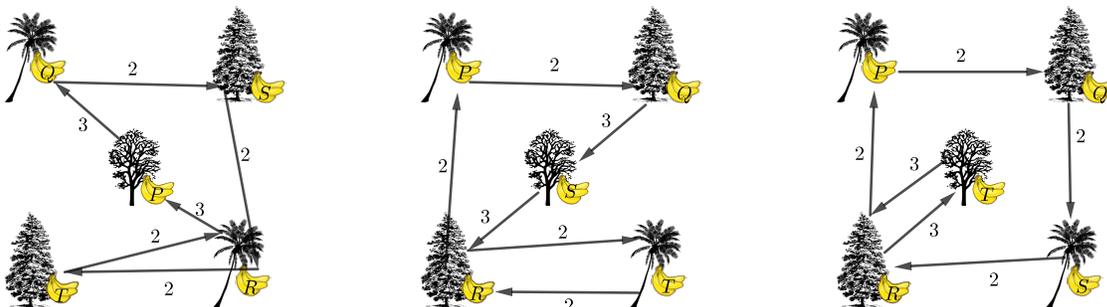

On the next page we will show why it is not possible to have a 14 second route if the bananas on the maple tree are of type R or Q.

If the bananas on the maple are of type R then there are four swings to or from the maple and the minimum time is now $4(3) + 2(2) = 16$. This is more than 14 seconds. Therefore, if the bananas on the maple are of type R, we cannot achieve the minimum time of 14 seconds.

If the bananas on the maple tree is are of type Q, then to obtain a total time of 14 seconds, the monkey must take two seconds to swing from each of S to R, R to T and R to P.
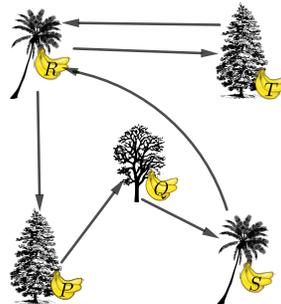
This means that the bananas $S, T,$ and P are all palm trees or all pine trees, which contradicts the initial situation.

Another way of thinking about this is that since the monkey will swing from each of S to R, R to T and R to P then at least one of the swings must take 7 s and the minimum time is $2(3) + 3(2) + 7 = 19$ seconds. This is more than 14 seconds. Therefore, if the bananas on the maple are of type Q, we cannot achieve the minimum time of 14 seconds

In conclusion, the bananas on the maple tree could be type $P, S,$ or T.

Applications to Computer Science

This problem involves finding the best, or optimal, solution to a problem. Computers are often used to find the maximum or minimum value of some measurement. In this case, we might think of the trees as applications on a touch screen and the monkey swinging as the movement of a human finger from one application to another. A user interface designer might be interested in how to arrange the applications for a common sequence of operations so as to require as little time as possible.