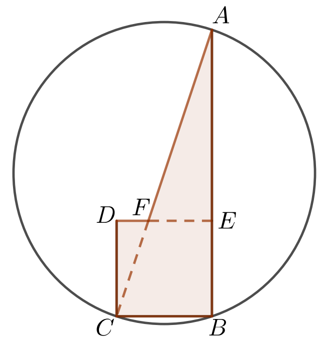
# Problem of the Week

## Problem E and Solution

## What's Left?




### Problem

$BCDE$ is a square with sides of length 20 cm. $BE$ is extended to $A$ such that the area of $\triangle ABC$ is twice the area of the square. The figure $ABCDF$ is enclosed in a circle with diameter $AC$ and point $B$ on the circumference of the circle. See the diagram to the right. Determine the area inside the circle but outside figure $ABCDF$.

## Solution

To find the area of the unshaded region, we need to find the area of the circle and subtract the area of the shaded figure $ABCDF$. To find the area of the circle we need the radius, which is half the length of diameter $AC$. To find the area of the shaded figure, we need to find the areas of square $BCDE$ and $\triangle AEF$. We will need to find the length of $EF$.

$$\begin{aligned}
\text{Area of Square } BCDE &= 20 \times 20 = 400 \text{ cm}^2 \\
\text{Area of } \triangle ABC &= 2 \times \text{Area of Square } BCDE = 800 \text{ cm}^2 \\
\text{But Area of } \triangle ABC &= (BC)(AB) \div 2 \\
\therefore 800 &= (20)(AB) \div 2 \\
AB &= 80 \text{ cm} \\
\text{Then } AE &= AB - BE = 80 - 20 = 60 \text{ cm.}
\end{aligned}$$

Since $\angle AEF = \angle ABC = 90^\circ$ and $\angle FAE = \angle CAB$, then $\triangle AEF \sim \triangle ABC$.

$$\begin{aligned}
\therefore \frac{AE}{AB} &= \frac{EF}{BC} \\
\frac{60}{80} &= \frac{EF}{20} \\
EF &= 15 \text{ cm}
\end{aligned}$$

$$\begin{aligned}
\text{Since } \triangle ABC \text{ is right angled, } AC^2 &= BC^2 + AB^2 \\
&= 20^2 + 80^2 \\
&= 6800 \\
AC &= 20\sqrt{17} \text{ cm}
\end{aligned}$$

But $AC$ is the diameter of the circle so the radius is $10\sqrt{17}$.

$$\begin{aligned}
\text{Unshaded Area} &= \text{Area of Circle} - \text{Area of } ABCDF \\
&= \text{Area of Circle} - (\text{Area of Square } BCDE + \text{Area of } \triangle AEF) \\
&= \pi(10\sqrt{17})^2 - [(20 \times 20) + (15 \times 60 \div 2)] \\
&= 1700\pi - 400 - 450 \\
&= (1700\pi - 850) \text{ cm}^2
\end{aligned}$$

The area inside the circle but outside the shaded figure is $(1700\pi - 850)$ cm$^2$ or approximately 4 491 cm$^2$.