### Problem of the Week
#### Problem A and Solution
#### What’s My Number?

**Problem**

<table>
<thead>
<tr>
<th>I am a 3-digit even number.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The sum of my three digits is 20.</td>
</tr>
<tr>
<td>I am greater than (40 \times 10).</td>
</tr>
<tr>
<td>I am less than (1000 \div 2).</td>
</tr>
<tr>
<td>What number am I?</td>
</tr>
</tbody>
</table>

**Solution**

Since \(40 \times 10 = 400\), we know the number is greater than 400. Since the number is even, the smallest possible number is 402.

Since \(1000 \div 2 = 500\), we know the number is less than 500. Since the number is even, the largest possible number is 498.

So we are looking for an even number between 402 and 498, inclusive. We could check each of the numbers in that range, to see which one has digits that add up to 20. However, that would mean checking 49 numbers. It would be better to reduce the range of numbers to check, if possible.

Here is one way of thinking about the solution:

Since the possible numbers are from 402 to 498, the first digit of the number is 4. Since the sum of the digits is 20, then the middle and last digit must sum to \(20 - 4 = 16\). Since the number is even, its last digit is 0, 2, 4, 6, or 8.

- If the last digit is 0, then the middle digit must be \(16 - 0 = 16\), which is not a digit from 0 to 9.
- If the last digit is 2, then the middle digit must be \(16 - 2 = 14\), which is not a digit from 0 to 9.
- If the last digit is 4, then the middle digit must be \(16 - 4 = 12\), which is not a digit from 0 to 9.
- If the last digit is 6, then the middle digit must be \(16 - 6 = 10\), which is not a digit from 0 to 9.
- If the last digit is 8, then the middle digit must be \(16 - 8 = 8\), which is a valid digit.

We have examined all possible cases and the only number satisfying all of the conditions is 488. The number we are looking for is 488.
Teacher’s Notes

Here is another way of thinking about the solution:

Since the numbers from 402 to 498 all start with the digit 4, and the sum of all three digits must be 20, then the sum of the last two digits of the number must be $20 - 4 = 16$.

Since the largest possible digit is 9, then the other digit of the number must be at least $16 - 9 = 7$. So each of the last two digits must be in the range 7 to 9.

Since we are only considering even numbers, then the solution must be a number that ends with an 8.

So we can look at even numbers, that start with a 4, end with an 8, and where the middle digit is at least 7. The possibilities are: 478, 488, and 498. Now we can check the sum of the digits of these numbers:

- $4 + 7 + 8 = 19$
- $4 + 8 + 8 = 20$
- $4 + 8 + 9 = 21$

So the only number that satisfies all of the requirements is 488.