Problem of the Week
Problem B and Solution
Luck of the Draw

Problem
A deck of cards without jokers has 52 cards, with 13 cards in each of the four suits: spades, hearts, diamonds, and clubs. The 13 cards are 2, 3, . . . , 9, 10, Jack, Queen, King, and Ace.

a) What is the theoretical probability, $P$, of drawing an Ace (of any suit) from a full deck?

b) After drawing the first Ace, if that card is not replaced in the deck, what is the probability, $R$, of drawing a second Ace from the remaining cards?

c) How would your answer to part b) change if the first card had been replaced in the deck before the second card was drawn?

d) The probability of two desired cards being drawn one after the other is the product of the two individual probabilities. For example, the probability of drawing two Aces when the first card is replaced in the deck is $P \times P$, with $P$ as in part a). The probability of drawing two Aces when the first card is not replaced in the deck is $P \times R$, with $P$ as in part a) and $R$ as in part b).

What is the probability of drawing the Ace of hearts, and then a 6 of any suit, if the first card is NOT replaced in the deck before the second card is drawn?

Solution
a) Since there are 4 Aces among the 52 cards, the probability of drawing an Ace is $P = 4 \div 52 = \frac{1}{13}$.

b) After one Ace is removed, there remain 51 cards with 3 Aces among them. Thus the probability of drawing a second Ace is $R = 3 \div 51 = \frac{1}{17}$.

c) If the first card had been replaced, there would once again have been 52 cards, 4 of which are Aces. So, the probability of drawing the second Ace would again be $P = \frac{1}{13}$.

d) Since there is one Ace of hearts, the probability of that Ace first being drawn is $\frac{1}{52}$. Since there are four 6’s in the remaining 51 cards, the probability of then drawing any 6 is $\frac{4}{51}$. Thus the probability of both draws being successful is $\frac{1}{52} \times \frac{4}{51} = \frac{1}{663}$.