Problem of the Week
Problem C and Solution
Count Down to Zero

Problem
Every year there is a countdown to the New Year in Problemville. The timer starts at 20 and counts down to 0. The display for the digits on the timer is made up of seven segments that are either lit or unlit. When the digit 8 is displayed, all seven segments are lit. When the digit 1 is displayed, only two segments are lit and five segments are unlit. In changing from digit to digit, a segment can change from lit to unlit, from unlit to lit, or could remain unchanged. For example, in changing from 5 to 4, three of the segments that were lit stay lit, one segment that was unlit stays unlit, one segment that was unlit becomes lit, and two of the segments that were lit become unlit. Therefore, there is a total of three changes of state when the timer changes from 5 to 4. In counting down from 20 to 0, how many changes of state are there? In other words, determine the number of times segments are turned from unlit to lit plus the number of times segments are turned from lit to unlit. (Note that, in changing from 10 to 9, the left digit is turned completely off.)

Solution

1. Consider the changes to the tens digit as a result of counting from 20 to 0.

   ![Image of numbers 20 and 19]

   When the counter changes from 20 to 19, the tens digit, 2, changes to a 1. As a result of this change, four segments change from lit to unlit, one segment changes from unlit to lit, and the other two segments remain unchanged. There are a total of $4 + 1 = 5$ state changes.

   ![Image of numbers 10 and 9]

   When the counter changes from 10 to 9, the tens digit, 1, turns off. As a result of this change, two segments become unlit, and the remaining five segments remain unchanged. There are 2 state changes to the tens digit when counting from 10 to 9.

   During the entire process of counting from 20 to 0, there are a total of $5 + 2 = 7$ changes in the state of the segments used for the tens digit.

2. Consider the changes to the units digit as a result of counting from 20 to 0.

   The total number of changes in the units digit going from 20 to 10 is exactly the same as the number of changes in the units digit going from 10 to 0. We will count the number of changes going from 20 to 10 and double our result.
In going from 20 to 19, the units digit changes from 0 to 9. One segment goes from lit to unlit, one segment goes from unit to lit, the remaining five segments remain unchanged. There is a total of 2 changes.

In going from 19 to 18, the units digit changes from 9 to 8. One segment goes from unlit to lit and the remaining six segments remain unchanged. There is 1 change.

In going from 18 to 17, the units digit changes from 8 to 7. Four segments go from lit to unlit and the remaining three segments remain unchanged. There is a total of 4 changes.

In going from 17 to 16, the units digit changes from 7 to 6. Four segments go from unlit to lit, one segment goes from lit to unlit, and the other two segments remain unchanged. There is a total of 5 changes.

In going from 16 to 15, the units digit changes from 6 to 5. One segment goes from lit to unlit, and the other six segments remain unchanged. There is 1 change.

In going from 15 to 14, the units digit changes from 5 to 4. Two segments go from lit to unlit, one segment goes from unlit to lit and the other four segments remain unchanged. There is a total of 3 changes.

In going from 14 to 13, the units digit changes from 4 to 3. One segment goes from lit to unlit, two segments go from unlit to lit and the other four segments remain unchanged. There is a total of 3 changes.

In going from 13 to 12, the units digit changes from 3 to 2. One segment goes from lit to unlit, one segment goes from unlit to lit and the other five segments remain unchanged. There is a total of 2 changes.

In going from 12 to 11, the units digit changes from 2 to 1. Four segments go from lit to unlit, one segment goes from unlit to lit and the other two segments remain unchanged. There is a total of 5 changes.

In going from 11 to 10, the units digit changes from 1 to 0. Four segments go from unlit to lit and the other three segments remain unchanged. There is a total of 4 changes.

So, in counting from 20 to 10 there is a total of \(2 + 1 + 4 + 5 + 1 + 3 + 3 + 2 + 5 + 4 = 30\) changes to the units digit. Counting from 10 to 0 will produce the same number changes to the units digit that occurred in going from 20 to 10 so we must count another 30 changes.

In total, there are 7 changes in the tens digit and 60 changes in the units digit, for a total of 67 segment changes in counting down from 20 to 0.