Problem of the Week

Problem B

In an Orderly Fashion

a) When we write the year 2021, we are writing two consecutive two-digit numbers (20 and 21). Find all the other years from 1000 to 2021 that are made up of two consecutive two-digit numbers written in order, and add them to the table below.

b) Find the sum of the consecutive two-digit numbers for each year from part a), and add this to the table below. For example, for 2021, the sum is $20 + 21 = 41$. Describe the pattern formed by these sums.

c) Find the product of the consecutive two-digit numbers for each of the first 5 years in the table. Then find the differences of these products, in order. For example, $10 \times 11 = 110$ and $11 \times 12 = 132$. The difference is $132 - 110 = 22$.
You will have five products and four differences.

(i) What pattern is formed by the differences?

(ii) Use this pattern to find the remaining products, without multiplying.

d) What sequence of numbers can you form by combining the numbers in the sum column and the difference column?

Year	Sum	Product	Difference
1011	$10 + 11 = 21$	$10 \times 11 = 110$	—
1112	$11 + 12 = 23$	$11 \times 12 = 132$	$132 - 110 = 22$
2021	$20 + 21 = 41$	$20 \times 21 = 420$