Problem of the Week

Problem B and Solution

'Place' Value

Problem

Complete the puzzles below by entering the digits from 1 through 9 in the blank boxes to make all the horizontal and vertical statements true. In each puzzle, use each digit just once and do the operations in the order that they appear, from left to right and from top to bottom.

a)

	+	6	x		=	56
+		+		−		
	−		+		=	12
÷		x		x		
	+		−		=	4
=		=		=		
5		50		3		

b)

	+		x		=	52
+		−		+		
	−		+		=	14
÷		x		÷		
7	+		÷		=	5
=		=		=		
2		21		5		

In puzzle a), the location of the digit 6 has been given, so the eight empty boxes each contain a different digit from 1 through 9, other than 6. In puzzle b), the location of the digit 7 has been given, so the eight empty boxes each contain a different digit from 1 through 9, other than 7.

Solution

First, we will give you the final grids.

a)

1	+	6	x	8	=	56
+		+		−		
9	−	4	+	7	=	12
÷		x		x		
2	+	5	−	3	=	4
=		=		=		
5		50		3		

b)

5	+	8	x	4	=	52
+		−		+		
9	−	1	+	6	=	14
÷		x		÷		
7	+	3	÷	2	=	5
=		=		=		
2		21		5		

On the next page we will give a solution for each grid.

For puzzle a):

Label the unknown values with the letters shown below.

a	+	6	×	b	=	56
+		+		−		
c	−	d	+	e	=	12
÷		×		×		
f	+	g	−	h	=	4
=		=		=		
5		50		3		

Let's start with g. From the second column, we see that g must be 1, 2, or 5, since these are the only digits that divide into 50.
If $g = 1$, then $6 + d$ must be 50. This is not possible since d must be a single digit.
If $g = 2$, then $6 + d$ must be 25. This is not possible since d must be a single digit.
If $g = 5$, then since $10 \times 5 = 50$, $6 + d$ must be 10, and so $d = 4$. This is possible.
Therefore $g = 5$ and $d = 4$. We add these to the grid.

a	+	6	×	b	=	56
+		+		−		
c	−	4	+	e	=	12
÷		×		×		
f	+	5	−	h	=	4
=		=		=		
5		50		3		

Next, we'll look at b. From the first row, we see that b must be 1, 2, 4, 7 or 8, since these are the only digits that divide 56. However, b cannot be 4 since $d = 4$. Also, b cannot be 1 or 2 because then $a + 6$ must be 56 or 28, which are both not possible since a must be a single digit. Therefore, $b = 7$ or $b = 8$.
If $b = 7$, then $a + 6$ must be 8, and so $a = 2$.
If $a = 2$, then $c = 3$ and $f = 1$ or $c = 8$ and $f = 2$.
However, if $c = 3$, then $e = 13$. Since 13 is not a digit, this is not possible.
If $c = 8$, then both $a = 2$ and $f = 2$, which is not possible. Therefore, b cannot be 7.
Therefore, $b = 8$ and $a + 6$ must be 7, and so $a = 1$. We add these to the grid.

1	+	6	×	8	=	56
+		+		−		
c	−	4	+	e	=	12
÷		×		×		
f	+	5	−	h	=	4
=		=		=		
5		50		3		

Next, we'll look at c. From the first column, we know that $1 + c$ is a multiple of 5. This means c is either 4 or 9. Since $d = 4$, c cannot be 4.
Therefore, $c = 9$.
Since $c = 9$, then since $10 \div 2 = 5$ we must have $f = 2$.
From the second row, since $c = 9$, then $e = 7$.
From the third row, since $f = 2$, then $h = 3$ and the grid is now complete.

1	+	6	×	8	=	56
+		+		−		
9	−	4	+	7	=	12
÷		×		×		
2	+	5	−	3	=	4
=		=		=		
5		50		3		

For puzzle b):

Label the unknown values with the letters shown below.

A	+	B	×	C	=	52
+		−		+		
D	−	E	+	F	=	14
÷		×		÷		
7	+	G	÷	H	=	5
=		=		=		
2		21		5		

Let's start with G. From the second column, G must be a 1, 3 or 7 since they are the only single digits that divide into 21.
If $G = 1$, then $B - E$ must be 21. This is not possible since B and E must be single digits.
Also, G cannot be 7 since the digit 7 is already in the grid.
Therefore, $G = 3$. Looking at the third row, since $G = 3$ and $10 \div 2 = 5$, then $H = 2$. We add these to the grid.

A	+	B	×	C	=	52
+		−		+		
D	−	E	+	F	=	14
÷		×		÷		
7	+	3	÷	2	=	5
=		=		=		
2		21		5		

Next, we'll look at C. From the first row, C must be 1, 2, or 4, since they are the only single digits that divide into 52.
If $C = 1$, then $A + B$ must be 52. This is not possible since A and B must be single digits.
Also, C cannot be 2 since the digit 2 is already in the grid.
Therefore, $C = 4$.
From the third column, since $10 \div 2 = 5$, then $4 + F$ must be 10. This means $F = 6$. We add this to the grid.

A	+	B	×	4	=	52
+		−		+		
D	−	E	+	6	=	14
÷		×		÷		
7	+	3	÷	2	=	5
=		=		=		
2		21		5		

The numbers we have not used yet are 1, 5, 8, and 9. Using the second column, we see $B - E$ must be 7 since $21 \div 3 = 7$. The only two remaining numbers that can make this equation true are $B = 8$ and $E = 1$.
From the first row, since $13 \times 4 = 52$, $A + 8$ must be 13, and so $A = 5$. This leaves $D = 9$ and the grid is now complete.

5	+	8	×	4	=	52
+		−		+		
9	−	1	+	6	=	14
÷		×		÷		
7	+	3	÷	2	=	5
=		=		=		
2		21		5		