

Problem of the Week

Problem C and Solution

Making a Splash!

Problem

A family has decided that they want to have a pool installed in their backyard. The city they live in has a by-law which states, “No pool may occupy more than 20% of the total area of the backyard in which it is to be installed.”

Their backyard is rectangular and measures 24 m by 18 m. The family creates a design in which they divide their backyard into a 3 by 3 grid of nine identical rectangles. The pool will be circular with the circumference of the pool passing through the four vertices of the middle rectangle. The middle rectangle will be completely covered by the pool. The distance across the pool through its centre will equal the length of the diagonal of the middle rectangle. Their plan is illustrated on the diagram above.

Should their plan be approved by the city?

Solution

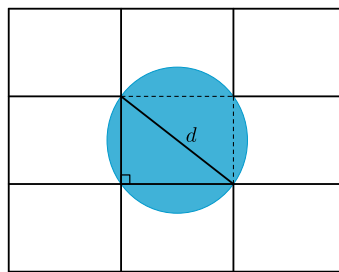
The dimensions of the backyard are 24 m by 18 m, so the area of the entire backyard is $24 \times 18 = 432 \text{ m}^2$. To comply with the by-law, the area of the pool cannot be more than 20% or $\frac{1}{5}$ of the total area of the backyard.

That is, the area cannot exceed $\frac{1}{5}$ of 432, which is equal to $\frac{1}{5} \times 432 = 86.4 \text{ m}^2$.

Since we know the length of the backyard is 24 m and the backyard is 3 rectangles long, the length of one rectangle is $24 \div 3 = 8 \text{ m}$. Similarly, since we know the width of the backyard is 18 m and the backyard is 3 rectangles across, the width of one rectangle is $18 \div 3 = 6 \text{ m}$.

Two adjacent sides of the middle rectangle and a diagonal form a right-angled triangle. Let d represent the length of the hypotenuse of the right-angled triangle with sides 8 m and 6 m. We know that d is also the length of the diameter of the pool. Using the Pythagorean Theorem,

$$\begin{aligned}d^2 &= 8^2 + 6^2 \\&= 64 + 36 \\&= 100 \\d &= \sqrt{100}, \text{ since } d > 0 \\d &= 10 \text{ m}\end{aligned}$$



Therefore, the diameter of the pool is 10 m, so it follows that the radius of the pool will be $10 \div 2 = 5 \text{ m}$.

The area of the pool can be calculated using the formula $A = \pi r^2$ so $A = \pi \times (5)^2 = 25\pi \approx 78.5 \text{ m}^2$. Since $78.5 < 86.4$, the area of the pool is less than one-fifth of the total area of the backyard.

Therefore, their plan should be approved.