Problem of the Week
Problem C and Solution
This Product is a Mystery

Problem

The number $A8$ is a two-digit number with tens digit A and units (ones) digit 8. Similarly, $3B$ is a two-digit number with tens digit 3 and units digit B.

When $A8$ is multiplied by $3B$, the result is the four-digit number $C730$. If A, B, and C are each different digits from 0 to 9, determine the values of A, B, and C.

Solution

In a multiplication question there are three parts: the *multiplier*, *multiplicand* and *product*. In our problem, $A8$ is the multiplier, $3B$ is the multiplicand, and $C730$ is the product.

The units digit of the product $C730$ is 0. The units digit of a product is equal to the units digit of the result obtained by multiplying the units digits of the multiplier and multiplicand.

So $8 \times B$ must equal a number with units digit 0. The only choices for B are 0 and 5, since no other single digit multiplied by 8 produces a number ending in zero.

However, if $B = 0$, the units digit of the product is 0 and the remaining three digits of the product, $C73$, are produced by multiplying $3 \times A8$. But $3 \times A8$ produces a number ending in 4, not 3 as required. Therefore $B \neq 0$ and B must equal 5. So the multiplicand is 35.

Since $A8$ is a two-digit number, the largest possible value for A is 9. Since $98 \times 35 = 3430$, the largest possible value of C is 3. Also, the product $C730$ is a four-digit number so $C \neq 0$. Therefore, the only possible values for C are $1, 2,$ and 3. We will examine each possibility for C.

If $C = 1$, then $C730$ becomes 1730. We want $A8 \times 35 = 1730$. Alternatively, we want $1730 \div 35 = A8$. But $1730 \div 35 \doteq 49.4$, which is not a whole number. Therefore, $C \neq 1$.

If $C = 2$, then $C730$ becomes 2730. We want $A8 \times 35 = 2730$. Alternatively, we want $2730 \div 35 = A8$. Since $2730 \div 35 = 78$, which is a whole number. Therefore, $C = 2$ produces a valid value for A, namely $A = 7$.

If $C = 3$, then $C730$ becomes 3730. We want $A8 \times 35 = 3730$. Alternatively, we want $3730 \div 35 = A8$. But $3730 \div 35 \doteq 106.6$, which is not a whole number. Therefore, $C \neq 3$.

We have examined every valid possibility for C and found only one solution. Therefore, $A = 7$, $B = 5$ and $C = 2$ is the only valid solution. We can easily verify that $78 \times 35 = 2730$.