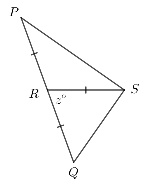


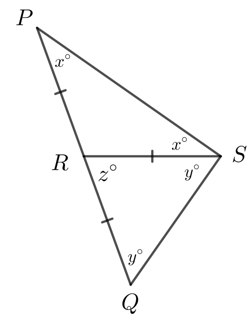
Problem of the Week
Problem D and Solution
Angled II

Problem

In $\triangle PQS$ above, R lies on PQ such that $PR = RQ = RS$ and $\angle QRS = z^\circ$.
Determine the measure of $\angle PSQ$.

Solution

Solution 1



In $\triangle PRS$, since $PR = RS$, $\triangle PRS$ is isosceles and $\angle RPS = \angle RSP = x^\circ$.

Similarly, in $\triangle QRS$, since $RQ = RS$, $\triangle QRS$ is isosceles and $\angle RQS = \angle RSQ = y^\circ$.

Since PRQ is a straight line, $\angle PRS + \angle QRS = 180^\circ$. Since $\angle QRS = z^\circ$, we have $\angle PRS = 180 - z^\circ$.

The angles in a triangle sum to 180°, so in $\triangle PRS$

$$\begin{aligned} \angle RPS + \angle RSP + \angle PRS &= 180^\circ \\ x^\circ + x^\circ + 180 - z^\circ &= 180^\circ \\ 2x &= z \\ x &= \frac{z}{2} \end{aligned}$$

The angles in a triangle sum to 180°, so in $\triangle QRS$

$$\begin{aligned} \angle RQS + \angle RSQ + \angle QRS &= 180^\circ \\ y^\circ + y^\circ + z^\circ &= 180^\circ \\ 2y &= 180 - z \\ y &= \frac{180 - z}{2} \end{aligned}$$

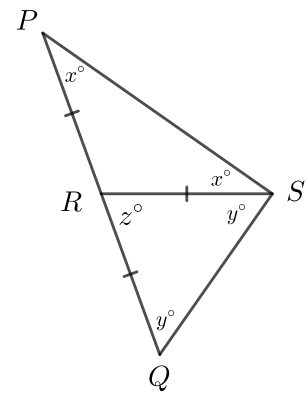
Then $\angle PSQ = \angle RSP + \angle RSQ = x^\circ + y^\circ = \frac{z}{2}^\circ + \left(\frac{180-z}{2}\right)^\circ = \left(\frac{180}{2}\right)^\circ = 90^\circ$.

Therefore, the measure of $\angle PSQ$ is 90°.

See Solution 2 for a more general approach to the solution of this problem.

It turns out that it is not necessary to determine expressions for x and y in terms of z to solve this problem.

Solution 2



In $\triangle PRS$, since $PR = RS$, $\triangle PRS$ is isosceles and $\angle RPS = \angle RSP = x^\circ$.

Similarly, in $\triangle QRS$, since $RQ = RS$, $\triangle QRS$ is isosceles and $\angle RQS = \angle RSQ = y^\circ$.

The angles in a triangle sum to 180°, so in $\triangle PQS$

$$\begin{aligned}\angle QPS + \angle PSQ + \angle PQS &= 180^\circ \\ x^\circ + (x^\circ + y^\circ) + y^\circ &= 180^\circ \\ (x^\circ + y^\circ) + (x^\circ + y^\circ) &= 180^\circ \\ 2(x^\circ + y^\circ) &= 180^\circ \\ x^\circ + y^\circ &= 90^\circ\end{aligned}$$

But $\angle PSQ = \angle RSP + \angle RSQ = x^\circ + y^\circ = 90^\circ$.

Therefore, the measure of $\angle PSQ$ is 90°.