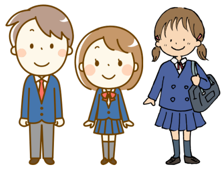

Problem of the Week
Problem E and Solution
Worth Checking

Problem

Students in three different classes wrote the same Calculus exam. The exam was marked out of 100. One class had 22 students in it. After writing the exam, their class average on the exam was reported as 87%. The second class had 27 students in it. After writing the exam, their class average on the exam was reported as 83%. The third class had 31 students in it. After writing the exam, their class average on the exam was reported as 81%.

Three students, Alf, Bet, and Tildi, discussed their results. Alf obtained a mark one less than Bet and Tildi obtained a mark one more than Bet. Upon reviewing their papers, Alf and Bet, both discovered addition errors on their papers. Both of their marks increased to 92. Tildi discovered that one of her questions had not been marked. This review resulted in her mark increasing to 92 as well. These changes resulted in the exam average for all of the students in the three classes combined changing to exactly 84%. What marks did Alf, Bet and Tildi originally have on their papers before the errors were corrected?

Solution

Solution 1

Let Bet's original mark be b. Then Alf's original mark is $b-1$ and Tildi's original mark is $b+1$.

The total number of marks for a class can be calculated by multiplying the number of students by the class average.

The total number of marks in the first class before the error was discovered was $22 \times 87 = 1914$. The total number of marks in the second class before the error was discovered was $27 \times 83 = 2241$. The total number of marks in the third class before the error was discovered was $31 \times 81 = 2511$.

The total of the marks from the three classes before correcting the errors was $1914 + 2241 + 2511 = 6666$. To correct the errors, we subtract the three incorrect marks from the total and add the three correct marks. The new total is therefore

$$6666 - (b-1) - b - (b+1) + 3 \times 92 = 6666 - 3b + 276 = 6942 - 3b \qquad (1)$$

The total number of students in the three classes combined was $22 + 27 + 31 = 80$. Since the average after correcting the three errors was exactly 84%, the total number of marks for the three classes was $80 \times 84 = 6720 \quad (2)$.

But (1) and (2) represent the same total. Therefore, $6942 - 3b = 6720$. It follows that $3b = 222$ and $b = 74$.

Since $b = 74$, $b - 1 = 73$ and $b + 1 = 75$.

Therefore, Alf's original mark was 73, Bet's original mark was 74 and Tildi's original mark was 75.

Solution 2

Let Bet's original mark be b. Then Alf's original mark is $b-1$ and Tildi's original mark is $b+1$.

The total number of marks for a class can be calculated by multiplying the number of students by the class average.

The total number of marks in the first class before the error was discovered was $22 \times 87 = 1914$. The total number of marks in the second class before the error was discovered was $27 \times 83 = 2241$. The total number of marks in the third class before the error was discovered was $31 \times 81 = 2511$.

The total of the marks from the three classes before correcting the errors was $1914 + 2241 + 2511 = 6666$.

The total number of students in the three classes combined was $22 + 27 + 31 = 80$. Since the average after correcting the three errors was exactly 84%, the total number of marks for the three classes was $80 \times 84 = 6720$.

The total mark change is then $6720 - 6666 = 54$.

But the total mark change can also be calculated by subtracting each of their old marks from 92 and then adding the three differences.

Therefore $[92 - (b-1)] + [92 - b] + [92 - (b+1)] = 54$. Then,

$$\begin{aligned} 92 - b + 1 + 92 - b + 92 - b - 1 &= 54 \\ 276 - 3b &= 54 \\ 222 &= 3b \\ 74 &= b \end{aligned}$$

Since $b = 74$, $b - 1 = 73$ and $b + 1 = 75$.

Therefore, Alf's original mark was 73, Bet's original mark was 74 and Tildi's original mark was 75.