



Problem of the Week

Problem E and Solution

Candies Anyone?

Problem

A jar contains only small red and small yellow candies. Another 30 red candies are added to the candies already in the jar so that one-third of the total number of candies in the jar are red candies. At this point, 30 yellow candies are added to the jar, and now three-tenths of the total number of candies in the jar are red candies. What fraction of the number of the candies originally in the jar were red candies?

Solution

Let r represent the number of red candies originally in the jar.

Let y represent the number of yellow candies originally in the jar.

Then $r + y$ represents the total number of candies originally in the jar.

After adding 30 red candies to the candies already in the jar, there are $(r + 30)$ red candies in the jar and a total of $(r + y + 30)$ candies in the jar. Now one-third of the candies in the jar are red candies, so

$$\begin{aligned}\frac{r + 30}{r + y + 30} &= \frac{1}{3} \\ 3(r + 30) &= 1(r + y + 30) \\ 3r + 90 &= r + y + 30 \\ 2r + 60 &= y\end{aligned}\tag{1}$$

After adding 30 yellow candies to the candies in the jar, there are $(r + 30)$ red candies in the jar and a total of $(r + y + 60)$ candies in the jar. Now three-tenths of the candies in the jar are red candies, so

$$\begin{aligned}\frac{r + 30}{r + y + 60} &= \frac{3}{10} \\ 10(r + 30) &= 3(r + y + 60) \\ 10r + 300 &= 3r + 3y + 180 \\ 7r - 3y &= -120\end{aligned}\tag{2}$$

Substituting (1) into (2),

$$\begin{aligned}7r - 3(2r + 60) &= -120 \\ 7r - 6r - 180 &= -120 \\ r &= 60\end{aligned}$$

Substituting $r = 60$ into (1), we get $y = 180$.

Therefore, there were originally 60 red and 180 yellow candies in the jar, and

$\frac{60}{60 + 180} = \frac{60}{240} = \frac{1}{4}$ of the candies originally in the jar were red.