Problem of the Week

Problem E

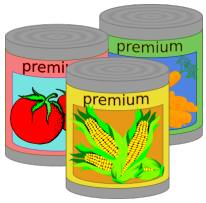
Can Drive On!

Bryn observed a real need in her community for canned food at the local food bank. On April 15, she decided to start a canned food drive, and hoped to collect 4000 cans of food by the end of June. She posted flyers and spread the word. She made the following observations.

Day Number	Total Number of Cans Collected Since Beginning of Drive	Increase from Previous Day
1	2	
2	5	3
3	12	7
4	23	11
5	38	15

Bryn noticed that the increases from one day to the next form an arithmetic sequence with first term 3 and common difference 4. (An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, $3, 5, 7, 9$ is an arithmetic sequence with four terms and common difference 2.)

Assuming that the pattern of daily increases continues, how many days would it take to collect at least 4000 cans of food?



The information on the next page *may* be helpful in solving the problem.

The sequence $3, 5, 7, 9$ is an arithmetic sequence with four terms and common difference 2. The term in position n is denoted t_n. For example, we say that $t_1 = 3$. The subscript 1 is the position of the term in the sequence and 3 is the value of the term.

The general term of an arithmetic sequence is $t_n = a + (n-1)d$, where a is the first term, d is the common difference, and n is the term number.

The sum, S_n, of the first n terms of an arithmetic sequence can be found using either $S_n = \frac{n}{2}(2a + (n-1)d)$ or $S_n = n\left(\frac{t_1 + t_n}{2}\right)$, where t_1 is the first term of the sequence and t_n is the n^{th} term of the sequence.

For example, for the arithmetic sequence $3, 5, 7, 9$, we have $a = t_1 = 3$, $d = 2$, $t_4 = 9$, and $S_4 = 3 + 5 + 7 + 9 = 24$.

Also,

$$\begin{aligned} \frac{4}{2}(2a + (4-1)d) &= \frac{4}{2}(2(3) + (4-1)2) \\ &= 2(12) \\ &= 24 \\ &= S_4 \end{aligned}$$

And,

$$\begin{aligned} 4\left(\frac{t_1 + t_4}{2}\right) &= 4\left(\frac{3+9}{2}\right) \\ &= 4(6) \\ &= 24 \\ &= S_4 \end{aligned}$$