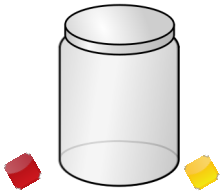

Problem of the Week
Problem E and Solution
Candies Anyone?

Problem

A jar contains only small red and small yellow candies. Another 30 red candies are added to the candies already in the jar so that one-third of the total number of candies in the jar are red candies. At this point, 30 yellow candies are added to the jar, and now three-tenths of the total number of candies in the jar are red candies. What fraction of the number of the candies originally in the jar were red candies?

Solution

Let r represent the number of red candies originally in the jar.
Let y represent the number of yellow candies originally in the jar.
Then $r + y$ represents the total number of candies originally in the jar.

After adding 30 red candies to the candies already in the jar, there are $(r + 30)$ red candies in the jar and a total of $(r + y + 30)$ candies in the jar. Now one-third of the candies in the jar are red candies, so

$$\begin{aligned} \frac{r+30}{r+y+30} &= \frac{1}{3} \\ 3(r+30) &= 1(r+y+30) \\ 3r+90 &= r+y+30 \\ 2r+60 &= y \qquad (1) \end{aligned}$$

After adding 30 yellow candies to the candies in the jar, there are $(r + 30)$ red candies in the jar and a total of $(r + y + 60)$ candies in the jar. Now three-tenths of the candies in the jar are red candies, so

$$\begin{aligned} \frac{r+30}{r+y+60} &= \frac{3}{10} \\ 10(r+30) &= 3(r+y+60) \\ 10r+300 &= 3r+3y+180 \\ 7r-3y &= -120 \qquad (2) \end{aligned}$$

Substituting (1) into (2),

$$\begin{aligned} 7r-3(2r+60) &= -120 \\ 7r-6r-180 &= -120 \\ r &= 60 \end{aligned}$$

Substituting $r = 60$ into (1), we get $y = 180$.

Therefore, there were originally 60 red and 180 yellow candies in the jar, and $\frac{60}{60+180} = \frac{60}{240} = \frac{1}{4}$ of the candies originally in the jar were red.