Problem of the Week
Problem E and Solution
What are the Possibilities?

Problem
Determine all values of $x$ that satisfy the equation \((x^2 - 5x + 5)^{x^2 + 4x - 60} = 1\).

Solution
Let’s consider the ways that an expression of the form \(a^b\) can be 1:

- **The base, \(a\), is 1.**
  In this case, the exponent can be any value and we need to solve \(x^2 - 5x + 5 = 1\).
  \[
  x^2 - 5x + 5 = 1 \\
  x^2 - 5x + 4 = 0 \\
  (x - 4)(x - 1) = 0
  \]
  So \(x = 4\) or \(x = 1\).

- **The exponent, \(b\), is 0.**
  In this case, the base can be any number other than 0 and we need to solve \(x^2 + 4x - 60 = 0\).
  \[
  x^2 + 4x - 60 = 0 \\
  (x - 6)(x + 10) = 0
  \]
  So \(x = 6\) or \(x = -10\).
  When \(x = 6\), the base is \(6^2 - 5(6) + 5 = 11 \neq 0\). That is, when \(x = 6\), the base does not equal 0.
  When \(x = -10\), the base is \((-10)^2 - 5(-10) + 5 = 155 \neq 0\). That is, when \(x = -10\), the base does not equal 0.

- **The base, \(a\), is \(-1\) and the exponent, \(b\), is even.**
  We first need to solve \(x^2 - 5x + 5 = -1\).
  \[
  x^2 - 5x + 5 = -1 \\
  x^2 - 5x + 6 = 0 \\
  (x - 2)(x - 3) = 0
  \]
  So \(x = 2\) or \(x = 3\).
  When \(x = 2\), the exponent is \(2^2 + 4(2) - 60 = -48\), which is even.
  Therefore, when \(x = 2\), \((x^2 - 5x + 5)^{x^2 + 4x - 60} = 1\).
  When \(x = 3\), the exponent is \(3^2 + 4(3) - 60 = -39\), which is odd.
  Therefore, when \(x = 3\), \((x^2 - 5x + 5)^{x^2 + 4x - 60} = -1\). So \(x = 3\) is not a solution.

Therefore, the values of \(x\) that satisfy \((x^2 - 5x + 5)^{x^2 + 4x - 60} = 1\) are \(x = -10, x = 1, x = 2, x = 4\) and \(x = 6\). There are five values of \(x\) which satisfy the equation.