Problem of the Week
Problem E
Cupid's Arrow

At a Valentine’s dance, contestants can participate in a game. They are blindfolded and spun around. They then try to place an arrow on a white heart on a red gameboard. (See the diagram below.)

The heart was constructed by attaching two white semi-circles onto the hypotenuse of an isosceles right-angled triangle. Each semi-circle has the same diameter, equal to half the length of the hypotenuse. The heart was then pasted onto a large rectangular sheet of red paper such that the hypotenuse of the triangle is parallel to the base of the rectangle. We know the following measurements:

- The distance from the top of a semi-circle to the top of the rectangle is 25 cm.
- The distance from the bottom vertex of triangle to the bottom of the rectangle is 25 cm.
- The length of each equal side of the triangle is \(25\sqrt{2}\) cm.
- The length of the base of the rectangle is 1 m.

(The dashed lines, the length measurements and the right angle symbol will not actually be on the finished gameboard.)

If a contestant places their arrow randomly somewhere on the gameboard, what is the probability that it will land on the white heart?

NOTE: To solve the problem, it may be helpful to use the following fact:
In an isosceles triangle, the line joining the vertex opposite the unequal side to the midpoint of the unequal side will be perpendicular to the unequal side.