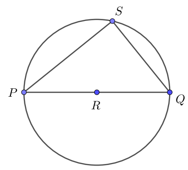


Problem of the Week
Problem E and Solution
Angled III

Problem

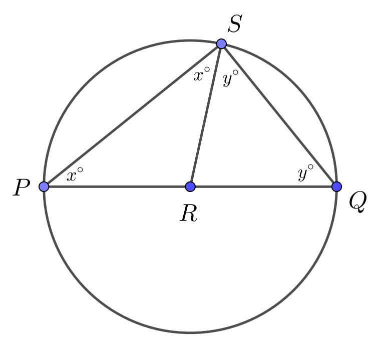
In the circle with centre R above, PQ is a diameter. Point S is a point on the circumference of the circle other than P or Q. Determine the measure of $\angle PSQ$.

Solution

Join S to the centre R. Since RP, RQ and RS are radii of the circle, $RP = RQ = RS$.

Since $RP = RS$, $\triangle PRS$ is isosceles and $\angle RPS = \angle RSP = x^\circ$.
Since $RQ = RS$, $\triangle QRS$ is isosceles and $\angle RQS = \angle RSQ = y^\circ$.

This new information is marked on the following diagram.



The angles in a triangle add to 180°, so in $\triangle PQS$

$$\begin{aligned}
\angle PSQ + \angle QPS + \angle PQS &= 180^\circ \\
(x^\circ + y^\circ) + x^\circ + y^\circ &= 180^\circ \\
2(x^\circ + y^\circ) &= 180^\circ \\
x^\circ + y^\circ &= 90^\circ
\end{aligned}$$

But $\angle PSQ = x^\circ + y^\circ$, so $\angle PSQ = 90^\circ$.

This result is often expressed as a theorem for circles:
An angle($\angle PSQ$) inscribed in a circle by a diameter (PQ) of the circle is 90°.