

Problem of the Week

Problem E and Solution

Roads All Around

Problem

Tima owns a triangular parcel of land that is created by three intersecting roads, as shown. Two of the roads meet at a right angle and two of the roads intersect at a 25° angle. If the perimeter of the triangular parcel of land is 1000 m, what is its area to the nearest 100 m^2 ?

Solution

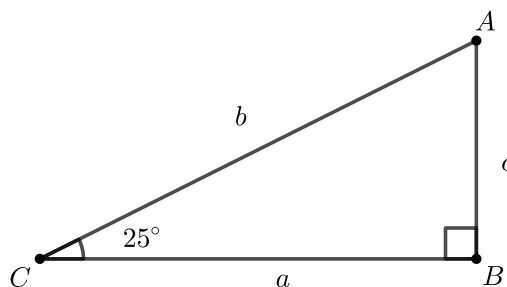
We will label the diagram as shown to the right.

We know the following:

$$a + b + c = 1000 \quad (1)$$

$$\frac{c}{b} = \sin(25^\circ), \text{ and so } c = b \sin(25^\circ) \quad (2)$$

$$\frac{a}{b} = \cos(25^\circ), \text{ and so } a = b \cos(25^\circ) \quad (3)$$



Substituting (2) and (3) into (1) we get:

$$b \cos(25^\circ) + b + b \sin(25^\circ) = 1000$$

$$b(\cos(25^\circ) + 1 + \sin(25^\circ)) = 1000$$

$$b = \frac{1000}{\cos(25^\circ) + 1 + \sin(25^\circ)}$$

$$b \approx 429.38 \text{ m}$$

Now, since $a = b \sin(25^\circ)$ and $c = b \cos(25^\circ)$, the area of the triangle is

$$\begin{aligned} \frac{ac}{2} &= \frac{(b \sin(25^\circ))(b \cos(25^\circ))}{2} \\ &= \frac{b^2 \sin(25^\circ) \cos(25^\circ)}{2} \\ &\approx \frac{(429.38)^2 \sin(25^\circ) \cos(25^\circ)}{2} \\ &= 35\,308.4 \text{ m}^2 \end{aligned}$$

Therefore, to the nearest 100 m^2 , the area of the triangle is $35\,300 \text{ m}^2$.