



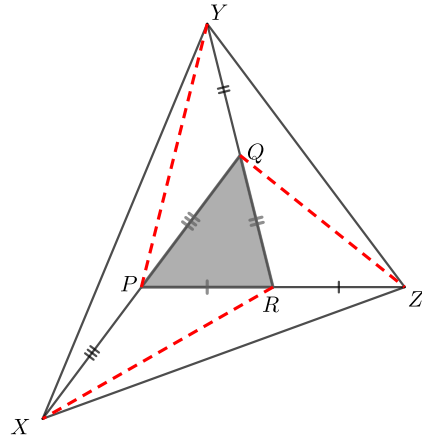
Problem of the Week

Problem E and Solution

Good Extensions

Problem

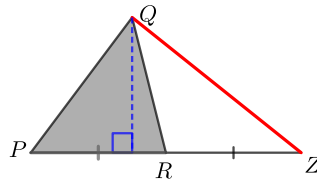
The sides of $\triangle PQR$ are extended to create $\triangle XYZ$ as follows: side RQ is extended to Y so that $RQ = QY$, side QP is extended to X so that $QP = PX$, and side PR is extended to Z so that $PR = RZ$. If the area of $\triangle XYZ = 1197$, determine the area of $\triangle PQR$.



Solution

On the above diagram, the lengths of the equal sides, $QP = PX$, $PR = RZ$, and $RQ = QY$, have been marked. Join P to Y , Q to Z , and R to X .

$\triangle PQR$ and $\triangle QRZ$ have a common altitude drawn from vertex Q to the line segment PZ . The triangles have equal base lengths, $PR = RZ$. Therefore, area $\triangle PQR = \text{area } \triangle QRZ = x$.



At this point we can proceed to look at various other triangles with equal areas. As you go through each pair of triangles, it may help to rotate the diagram so the bases of the triangles are always on the bottom.

$\triangle PQR$ and $\triangle PQY$ have the same height and equal base lengths ($RQ = QY$). Therefore, area $\triangle PQY = \text{area } \triangle PQR = x$.

$\triangle PQY$ and $\triangle PXY$ have the same height and equal base lengths ($QP = PX$). Therefore, area $\triangle PXY = \text{area } \triangle PQY = x$.

$\triangle PXR$ and $\triangle PQR$ have the same height and equal base lengths ($QP = PX$). Therefore, area $\triangle PXR = \text{area } \triangle PQR = x$.

$\triangle PXR$ and $\triangle RXZ$ have the same height and equal base lengths ($PR = RZ$). Therefore, area $\triangle RXZ = \text{area } \triangle PXR = x$.

$\triangle QRZ$ and $\triangle QYZ$ have the same height and equal base lengths ($RQ = QY$). Therefore, area $\triangle QYZ = \text{area } \triangle QRZ = x$.

Then,

$$\begin{aligned} \text{area } \triangle XYZ &= \text{area } \triangle PXY + \text{area } \triangle PQY + \text{area } \triangle PQR + \text{area } \triangle PXR + \text{area } \triangle RXZ \\ &\quad + \text{area } \triangle QRZ + \text{area } \triangle QYZ \\ &= x + x + x + x + x + x + x \end{aligned}$$

Therefore, $7x = 1197$ and $x = 171$.

Therefore, the area of $\triangle PQR$ is 171.