Problem of the Week Problem B and Solution Honesty - the Best Policy?

Problem

A reporter is doing interviews at a convention attended by 500 people.

- 40% of these people always tell the truth.
- 20 of them will always lie.
- The remaining people each sometimes tell the truth and sometimes lie.



- a) Suppose that she interviews a person at random.
 - (i) As a fraction, what is the probability that the person always lies?
 - (ii) As a fraction, what is the probability that the person always tells the truth?
- b) Suppose she interviews Joe Public, a person at the convention. The last time she interviewed him, he only told her the truth, so she knows he must be a person who always tells the truth or a person who sometimes tells the truth and sometimes lies. What is the probability that Joe Public always tells the truth?

Solution

- a) (i) Since there are 20 people who always lie out of 500 people, the probability she will at random interview someone who always lies is $\frac{20}{500}$ which can be reduced to $\frac{1}{25}$.
 - (ii) Since $40\% = \frac{40}{100}$ of the 500 people always tell the truth, the probability that the person always tells the truth is $\frac{40}{100}$. This can be reduced to $\frac{2}{5}$.
- b) Since Joe has told the truth so far, we know that he does not always lie. There are 20 people that always lie. Therefore, Joe is one of the remaining 500 - 20 = 480 people. Since 40% of the 500 people always tell the truth, that means 40 out of 100 people always tell the truth. We know $100 \times 5 = 500$. That means there are $40 \times 5 = 200$ people at the convention that always tell the truth. Therefore, the probability that Joe always tells the truth is $\frac{200}{480}$. This can be reduced to $\frac{5}{12}$.