Problem of the Week
Problem D and Solution
The Grandkids

Problem

My grandparents have four grandchildren. The product of the ages of the four grandchildren is 67 184. The youngest grandchild is younger than 10, and is also 30 years younger than the oldest grandchild. Determine all possibilities for the ages of my grandparents' grandchildren.

Solution

The first task is to factor 67 184. Since 67 184 is even, we can divide out powers of 2 to determine that $67\,184 = 2^4 \times 4199$. After some trial we discover that $67\,184 = 1 \times 2^4 \times 13 \times 17 \times 19$. Notice the inclusion of the number 1 as one of the factors. (One of the grandchildren could be 1.)

Since the youngest grandchild is younger than 10, from the factorization we can see that the youngest grandchild must be 1, 2, 4, or 8. We will consider these four cases for the age of the youngest grandchild separately.

If the youngest grandchild is 1, then the oldest grandchild would be $1 + 30 = 31$. Since there is no way to combine the other factors to create the age 31, this is not a possibility.

If the youngest grandchild is 2, then the oldest grandchild would be $2 + 30 = 32$. Since there is no way to combine the other factors to create the age 32, this is not a possibility.

If the youngest grandchild is 4, then the oldest grandchild would be $4 + 30 = 34$. Since $34 = 2 \times 17$, the other ages would have to be formed by combining one remaining power of 2, 13, and 19. That is, the other two ages could be either $2 \times 13 = 26$ and 19, or 13 and $2 \times 19 = 38$, or 2 and $13 \times 19 = 247$. Since $38 > 34$ and $247 > 34$ and the oldest grandchild is 34, the second and third possibilities are not valid. Therefore, we find that one possibility is that the grandchildren are 4, 34, 26 and 19.

If the youngest grandchild is 8, then the oldest grandchild would be $8 + 30 = 38$. Since $38 = 2 \times 19$, the other children would have to be 13 and 17. Therefore, we find that one more possibility is that the grandchildren are 8, 38, 13 and 17

Therefore, there are two possibilities for the ages of my grandparents' grandchildren. They could be 4, 19, 26 and 34, or they could be 8, 13, 17 and 38.