



Problem of the Week

Problem A and Solution

Adding and Subtracting

Problem

Follow the steps below.

Step 1: Pick two different three-digit numbers. Label the larger number A and the smaller number B .

Step 2: Subtract B from 999 and label the difference C .

Step 3: Add A and C and label the sum D .

Step 4: Subtract 1000 from D and label the difference E .

Step 5: Add 1 to E and label the sum F .

- What is the connection between the number F and the numbers A and B ?
- Try the same steps with different numbers. Do you think you will always get the same result? Why or why not?

Solution

- We can work through this procedure with any two, random, three-digit numbers. Let's try starting with 814 and 275.

Step 1: The larger number 814 is labelled A , and the smaller number 275 is labelled B .

Step 2: The difference $999 - B$ is $999 - 275 = 724$, and is labelled C .

Step 3: The sum $A + C$ is $814 + 724 = 1538$, and is labelled D .

Step 4: The difference $D - 1000$ is $1538 - 1000 = 538$, and is labelled E .

Step 5: The sum $E + 1$ is $538 + 1 = 539$, and is labelled F .

Notice that $814 - 275 = 539$. So $A - B = F$.

- If you try this procedure with any two three-digit numbers, it will always work out that $A - B = F$. We will show this using algebra.

Step 1: We choose three-digit numbers A and B so that $A \geq B$.

Step 2: We calculate $999 - B$, and label this C . That is, $C = 999 - B$.

Step 3: We add $A + C$. So, we calculate $A + 999 - B$, and label this D . That is, $D = A + 999 - B$.

Step 4: We subtract 1000 from D . So, we calculate $A + 999 - B - 1000$, and label this E . That is, $E = A + 999 - B - 1000$.

Step 5: We add 1 to E . So, we calculate $A + 999 - B - 1000 + 1$, and label this F . That is, $F = A + 999 - B - 1000 + 1$.

Now, we can simplify this expression to get $A - B + 999 - 1000 + 1 = A - B$, since $999 - 1000 + 1 = 0$. So, $F = A - B$.