



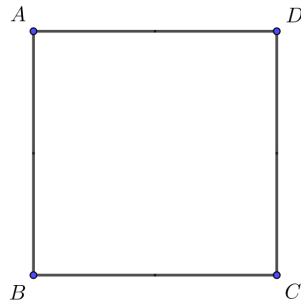
# Problem of the Week

## Problem A and Solution

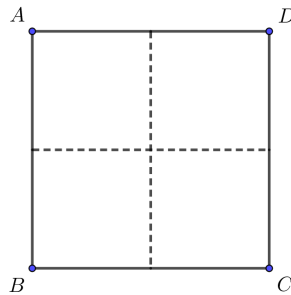
### Origami

#### Problem

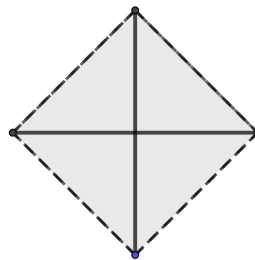
Laila starts with a square piece of paper. Starting at one corner and moving around the square, she labels the corners  $A$ ,  $B$ ,  $C$ , and  $D$ .



Laila folds the paper in half, by folding side  $AB$  onto side  $DC$ , to form a rectangle. She opens up the paper and folds it again to form another rectangle by folding side  $AD$  onto side  $BC$ . When she opens up the paper this time, she sees two creases in the paper as shown below.



The centre of the square is the point where the two creases intersect. Now, she takes each corner of the square and folds the paper so that each corner touches the centre of the square. Folding all four corners in this way forms another smaller square made up of four triangular regions as shown below.



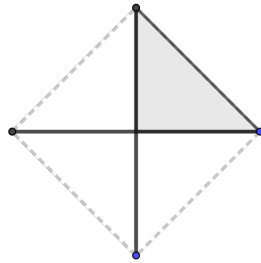
What fraction of the area of the original square is the area of this smaller square? Justify your answer.



### Solution

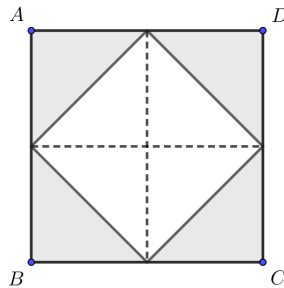
The smaller square has an area that is  $\frac{1}{2}$  the area of the original square. A justification of this is given below.

Consider the following image of the smaller square with only one of the four triangular regions shaded.



Underneath the shaded triangle is a region of the original square that is exactly the same size. That is true for all four of the triangles that were formed by having the corners meet at the centre of the original square.

Let's now shade all four of the triangular regions in the smaller square, and then open up the paper again.



We can make the following observations:

- The shaded triangles each have a matching unshaded triangle.
- The shaded triangles make up the area of the smaller square.
- The area of the original square is equal to the area of the shaded triangles plus the area of the smaller square. This means the area of the original square is 2 times the area of the smaller square.

It follows that the area of the smaller square is  $\frac{1}{2}$  the area of the original square.