



## Problem of the Week

### Problem A and Solution

### Cookies!

#### Problem

Benoit, Jin, Manuel, and Sarah shared a package of 30 cookies. After several days, all the cookies in the package were eaten by these four people and each person only ate whole cookies. Benoit recorded the following observations:

- Sarah ate twice as many cookies as Jin.
  - Manuel ate two-thirds as many cookies as Sarah.
  - Benoit ate half as many cookies as Manuel.
- (a) Who ate the most cookies? Justify your answer.
- (b) What fraction of the number of cookies eaten by Sarah is the number of cookies eaten by Benoit? Justify your answer.
- (c) Who ate the fewest cookies? Justify your answer.
- (d) Someone ate 6 cookies. This was not the person who ate the most or fewest cookies. How many cookies did each person eat? Justify your answer.

#### Solution

- (a) Since Sarah ate more cookies than Jin, we know that Jin did not eat the most cookies. Since Manuel ate fewer cookies than Sarah, we know that Manuel did not eat the most cookies. Since Benoit ate fewer cookies than Manuel, we know that Benoit did not eat the most cookies. Therefore, Sarah ate the most cookies.
- (b) Since Benoit ate half as many cookies as Manuel, and Manuel ate two-thirds as many cookies as Sarah, we need to calculate one-half of two-thirds, which is one-third. So Benoit ate one-third as many cookies as Sarah.
- (c) Since Sarah ate twice as many cookies as Jin, we can say that Jin ate one-half as many cookies as Sarah. From part (b) we know Benoit ate one-third as many cookies as Sarah. We also know that Manuel ate two-thirds as many cookies as Sarah. Since one-third is less than one-half, which is less than two-thirds, it follows that Benoit ate the fewest cookies.



- (d) Since we know that Sarah ate the most cookies, and Benoit ate the fewest cookies, then either Jin or Manuel ate 6 cookies. Let's guess that Jin ate 6 cookies.

In this case, from the first observation then Sarah ate  $2 \times 6 = 12$  cookies. We know that  $\frac{1}{3}$  of 12 is 4, so  $\frac{2}{3}$  of 12 is 8. From the second observation Manuel ate 8 cookies. From the third observation, Benoit ate  $\frac{1}{2}$  of 8, which is 4 cookies.

Since  $4 + 6 + 8 + 12 = 30$ , then it is possible that Benoit ate 4 cookies, Jin ate 6 cookies, Manuel ate 8 cookies, and Sarah ate 12 cookies.

We should also check to see if it is possible that Manuel was the person who ate 6 cookies. In this case, Benoit would have eaten  $\frac{1}{2}$  of 6, which is 3 cookies. From part (b) we noticed that Benoit ate  $\frac{1}{3}$  as many cookies as Sarah. This means we can say that Sarah ate 3 times as many cookies as Benoit, so Sarah would have eaten  $3 \times 3 = 9$  cookies. However, since Jin ate half as many cookies as Sarah, and one-half of 9 is not a whole number, that means Sarah could not have eaten 9 cookies. So Manuel could not be the person who ate 6 cookies.

We see that it must be the case that Benoit ate 4 cookies, Jin ate 6 cookies, Manuel ate 8 cookies, and Sarah ate 12 cookies.