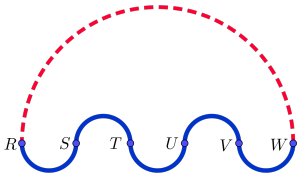

Problem of the Week
Problem C and Solution
Two Paths

Problem

Points R, S, T, U, V, and W lie in a straight line. There are two curved paths from R to W. The upper path is a semi-circle with diameter RW. The lower path is made up of five semi-circles with diameters RS, ST, TU, UV, and VW.

It is also known that the distance from R to W in a straight line is 1000 m, and $RS = ST = TU = UV = VW$.

Starting at the same time, John and Betty ride their bicycles along these paths from R to W. Betty follows the upper path and John follows the lower path. If they bike at the same speed, who will arrive at W first?

Solution

The circumference of a circle is found by multiplying its diameter by π. To find the circumference of a semi-circle, we divide its circumference by 2.

The length of the upper path is equal to half the circumference of a circle with diameter 1000 m. Therefore, the length of the upper path is equal to $\pi \times 1000 \div 2 = 500\pi$ m. (This is approximately 1570.8 m.)

Each of the semi-circles along the lower path have the same diameter. The diameter of each of these semi-circles is $1000 \div 5 = 200$ m. The length of the lower path is equal to half the circumference of five circles, each with diameter 200 m. Therefore, the distance along the lower path is equal to

$$5 \times (\pi \times 200 \div 2) = 5 \times (100\pi) = 500\pi \text{ m}$$

Since both John and Betty bike at the same speed and both travel the same distance, they will arrive at point W at the same time. The answer to the problem may surprise you.

Extension:

If you were to extend the problem so that Betty travels the same route but John travels along a lower path made up of 100 semi-circles of equal diameter from R to W, they would still both travel exactly the same distance, 500π m. Check it out!