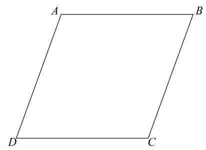


Problem of the Week
Problem D and Solution
This Angle Isn't Bad

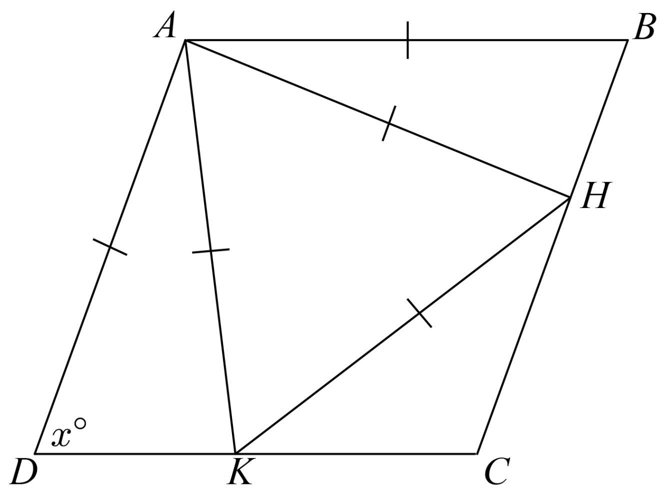
Problem

Ewan drew rhombus $ABCD$. Recall that a rhombus is a quadrilateral with parallel opposite sides, and all four sides of equal length. In Ewan's rhombus, H is on BC in between B and C, and K is on CD in between C and D, such that $AB = AH = HK = KA$.

Determine the measure, in degrees, of $\angle BAD$.

Solution

Since $ABCD$ is a rhombus, we know $AB = BC = CD = DA$. We're also given that $AB = AH = HK = KA$. Let $\angle ADK = x^\circ$.

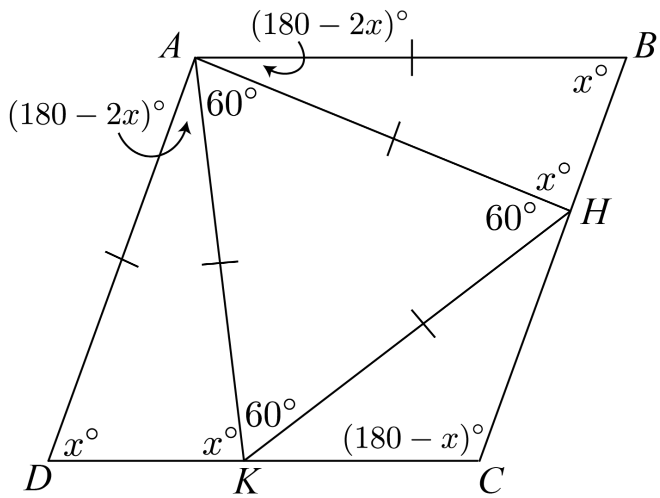


Since $AH = HK = KA$, $\triangle AHK$ is an equilateral triangle and each angle in $\triangle AHK$ is 60°. In particular, $\angle HAK = 60^\circ$.

In $\triangle ADK$, $AD = AK$ and so $\triangle ADK$ is isosceles. Therefore, $\angle AKD = \angle ADK = x^\circ$. Then $\angle DAK = (180 - 2x)^\circ$.

Since $ABCD$ is a rhombus, $AB \parallel CD$ and $\angle ADC + \angle BCD = 180^\circ$. It follows that $\angle BCD = (180 - x)^\circ$. But in the rhombus we also have $BC \parallel AD$ and $\angle BCD + \angle ABC = 180^\circ$. It follows that $\angle ABC = 180^\circ - (180 - x)^\circ = x^\circ$.

In $\triangle AHB$, $AH = AB$ and so $\triangle AHB$ is isosceles. Therefore, $\angle AHB = \angle ABH = x^\circ$. Then $\angle BAH = (180 - 2x)^\circ$.



Since $ABCD$ is a rhombus, $BC \parallel AD$, so

$$\begin{aligned}\angle BAD &= 180^\circ - \angle ABC \\ (180-2x)^\circ + 60^\circ + (180-2x)^\circ &= 180^\circ - x^\circ \\ (420-4x)^\circ &= (180-x)^\circ \\ 240^\circ &= (3x)^\circ \\ x^\circ &= 80^\circ\end{aligned}$$

It follows that

$$\begin{aligned}\angle BAD &= (180-x)^\circ \\ &= 180^\circ - 80^\circ \\ &= 100^\circ\end{aligned}$$

Therefore, $\angle BAD = 100^\circ$.