



Problem of the Week

Problem D and Solution

Coloured Areas

Problem

Chandra wishes to paint a sign. The sign is composed of three concentric semi-circles, creating an outer band, a middle band, and an inner semi-circle. The outer band is divided into five regions of equal area and will be painted blue. Each of these regions is labelled with a B . The middle band is divided into three regions of equal area and will be painted red. Each of these regions is labelled with an R . The inner semi-circle will be painted white. The diameter of the largest semi-circle is 10 m and the diameter of the middle semi-circle is 6 m. If the ratio of the area of one region marked with an R to the area of one region marked with a B is 5 : 6, what is the diameter of the inner semi-circle?

Solution

Since the area of a circle with radius r is πr^2 , the area of a semi-circle with radius r is $\frac{\pi r^2}{2}$. The large semi-circle has diameter 10 m and therefore has a radius of 5 m. Thus, the area of the large semi-circle is $\frac{\pi(5)^2}{2} = \frac{25\pi}{2}$ m². The middle semi-circle has diameter 6 m and therefore a radius of 3 m. Thus, the area of the middle semi-circle is $\frac{\pi(3)^2}{2} = \frac{9\pi}{2}$ m².

The area of the large semi-circle is made up of the areas of 5 regions marked with a B plus the area of the middle semi-circle. Therefore,

$$\begin{aligned} 5B + \frac{9\pi}{2} &= \frac{25\pi}{2} \\ 5B &= 8\pi \\ B &= \frac{8\pi}{5} \end{aligned}$$

Since ratio of the area of one red region marked with an R to the area of one blue region marked with a B is 5 : 6, we have $\frac{R}{B} = \frac{5}{6}$. And so,

$$\begin{aligned} R &= \frac{5B}{6} \\ &= \frac{5}{6} \left(\frac{8\pi}{5} \right) \\ &= \frac{4\pi}{3} \end{aligned}$$

Let the radius of the smallest semi-circle be r .

The area of the middle semi-circle is made up of the areas of 3 regions marked with an R plus the area of the smallest semi-circle. Therefore,

$$\frac{9\pi}{2} = 3R + \frac{\pi r^2}{2}$$

Since $R = \frac{4\pi}{3}$, we have

$$\frac{9\pi}{2} = 4\pi + \frac{\pi r^2}{2}$$

Therefore, $\frac{\pi}{2} = \frac{\pi r^2}{2}$ or $r^2 = 1$. Thus $r = 1$, since $r > 0$.

Therefore, the diameter of the smallest semi-circle is 2 m.