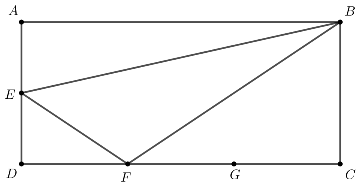


Problem of the Week

Problem D and Solution

The Whole Rectangle

Problem

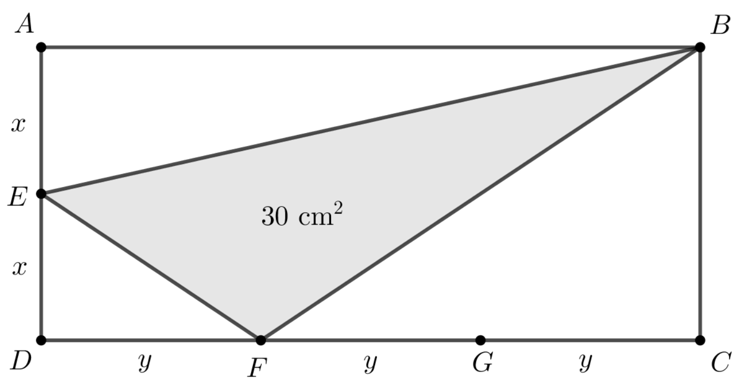
In the diagram, $ABCD$ is a rectangle. Points F and G are on DC (with F closer to D) such that $DF = FG = GC$. Point E is the midpoint of AD.

If the area of $\triangle BEF$ is 30 cm^2, determine the area of rectangle $ABCD$.

Solution

Let $DF = FG = GC = y$. Then $AB = DC = 3y$ and $FC = 2y$.

Since E is the midpoint of AD, let $AE = ED = x$. Then $AD = BC = 2x$.



We will formulate an equation connecting the areas of the four triangles inside the rectangle to the area of the entire rectangle.

$$\begin{aligned}
\text{Area } ABCD &= \text{Area } \triangle ABE + \text{Area } \triangle BCF + \text{Area } \triangle FDE + \text{Area } \triangle BEF \\
AD \times DC &= \frac{AE \times AB}{2} + \frac{BC \times FC}{2} + \frac{DF \times ED}{2} + 30 \\
(2x)(3y) &= \frac{x \times 3y}{2} + \frac{2x \times 2y}{2} + \frac{y \times x}{2} + 30 \\
6xy &= \frac{3xy}{2} + 2xy + \frac{xy}{2} + 30 \\
12xy &= 3xy + 4xy + xy + 60 \\
4xy &= 60 \\
xy &= 15
\end{aligned}$$

Therefore, the area of rectangle $ABCD$ is $AD \times DC = (2x)(3y) = 6xy = 6(15) = 90$ cm^2.