$$B > \frac{1}{n}\sum_{i=1}^{n} x_i$$

Problem of the Week
Problem E and Solution
Just an Average Sum

Problem

Faisal chooses four numbers. When each number is added to the mean (average) of the other three, the following sums are obtained: 25, 37, 43, and 51. Determine the mean of the four numbers Faisal chose.

Extra Problem: Can you interpret the picture puzzle above? You may need to research the meanings of some mathematical symbols used in the puzzle.

Solution

Let a, b, c, and d represent the four numbers. It is possible to precisely determine the four numbers, but the problem asks for only their average, which is $\frac{a+b+c+d}{4}$.

When the first number is added to the average of the other three numbers, the result is 25. Thus,

$$a + \frac{b+c+d}{3} = 25$$

which can be rewritten as

$$3a + b + c + d = 75 \quad (1)$$

When the second number is added to the average of the other three numbers, the result is 37. Thus,

$$b + \frac{a+c+d}{3} = 37$$

which can be rewritten as

$$a + 3b + c + d = 111 \quad (2)$$

When the third number is added to the average of the other three numbers, the result is 43. Thus,

$$c + \frac{a+b+d}{3} = 43$$

which can be rewritten as

$$a + b + 3c + d = 129 \quad (3)$$

When the fourth number is added to the average of the other three numbers, the result is 51. Thus,

$$d + \frac{a+b+c}{3} = 51$$

which can be rewritten as

$$a + b + c + 3d = 153 \quad (4)$$

Adding equations (1), (2), (3), and (4), we obtain $6a + 6b + 6c + 6d = 468$. Dividing this equation by 6 gives $a + b + c + d = 78$. It follows that $\frac{a+b+c+d}{4} = 19.5$.

Therefore, the average of the four numbers is 19.5.

Although it is not required, we could solve the system of equations to determine that the numbers are: -1.5, 16.5, 25.5, and 37.5.

Extra Problem Solution:

The notation $\frac{1}{n}\sum_{i=1}^{n} x_i$ is a mathematical short form which represents the average of the n numbers $x_1, x_2, \ldots, x_n$. So the picture puzzle can be interpreted as "Be greater than average".