



## Problem of the Week

### Problem E and Solution

### To Tell the Truth?



#### Problem

Four people, Nathaniel, Elizabeth, Libby, and Joel, each said two statements such that:

- one person lied in both statements;
- one person told the truth in both statements; and
- two people told the truth in one statement and a lie in the other statement.

Nathaniel said “Elizabeth lied exactly once” and “Joel lied twice”.

Elizabeth said “I never lie” and “Nathaniel never lied”.

Libby said “Joel lied twice” and “Elizabeth never lied”.

Joel said “Nathaniel lied twice” and “I never lie”.

Who lied twice? Who never lied? Who lied exactly once?

#### Solution

We will solve this problem by first figuring out who must have lied twice, and then figuring out who must have told the truth twice.

1. Who lied twice?
  - (a) Assume that Nathaniel lied twice. If so, “Elizabeth lied once” is a lie. Therefore, “Elizabeth never lied” or “Elizabeth lied twice.” But “Elizabeth never lied” cannot be true because Elizabeth says, “Nathaniel never lied.” This contradicts our assumption that Nathaniel lied twice. Also, “Elizabeth lied twice” cannot be true since that means Elizabeth and Nathaniel both lied twice and this contradicts the fact that only one person lied twice. Therefore, our assumption that Nathaniel lied twice is false.
  - (b) Assume that Elizabeth lied twice. If so, “Nathaniel never lied” is a lie. Then, Nathaniel lied twice or Nathaniel lied once. Nathaniel cannot have lied twice since both he and Elizabeth would have lied twice and this contradicts the fact that only one person lied twice. But “Nathaniel lied once” is also false since “Elizabeth lied exactly once” is a lie (we assumed she lied twice) and “Joel lied twice” is a lie because it contradicts the assumption that Elizabeth lied twice (and only one person can lie twice). Therefore, our assumption that Elizabeth lied twice is false.
  - (c) Assume that Libby lied twice. If so, “Elizabeth never lied” is a lie and “Joel lied twice” is a lie. Since “Elizabeth never lied” is a lie, then she lied twice or she lied once. But if Elizabeth lied twice our assumption that Libby lied twice cannot be true since only one person lied twice. If Elizabeth lied exactly once, then “I never lie” must be the lie and “Nathaniel never lied” must be true. But if Nathaniel never lied, then “Joel lied twice” must be true and this contradicts the fact that only one person can lie twice. Therefore, our assumption that Libby lied twice is false.



We are told that one person lied twice and none of Nathaniel, Elizabeth or Libby lied twice. Therefore, by elimination, Joel is the one who lied twice.

2. Who never lied?

- (a) Assume that Elizabeth never lied. Then her statement that “Nathaniel never lied” must be true. Then, there are then two people who never lied. This contradicts the fact that only one person never lied. Therefore, our assumption that Elizabeth never lied is false.
- (b) Assume that Libby never lied. Then her statement that “Elizabeth never lied” must be true. Then, there are then two people who never lied. This contradicts the fact that only one person never lied. Therefore, our assumption that Libby never lied is false.

Joel lied twice. Elizabeth and Libby lied. Therefore, by elimination, Nathaniel is the one who never lied. It then follows that Elizabeth and Libby each make one true statement and tell one lie.

We can now check our results.

Nathaniel never lied. Then his statements are both true. Elizabeth lied exactly once is true and Joel lied twice is true.

Elizabeth lied once. Then one of her statements is true and the other is a lie. Her statement that she never lies is a lie and her statement that Nathaniel never lies is true.

Libby lied once. Then one of her statements is true and the other is a lie. Her statement that Joel lied twice is true and her statement that Elizabeth never lied is a lie.

Joel lied twice. Then both of his statements are lies. Nathaniel lied twice is a lie. And his statement the he never lies is a lie.