Problem of the Week
Problem E and Solution
Pool Season

Problem

Philip plans to use small and large hoses to fill a swimming pool. They know that it takes four hours for nine large hoses to fill the pool. They also know that it takes eight hours for six small hoses to fill the pool.

How long will it take to fill the pool if Philip uses four large hoses and eight small hoses?

Solution

Solution 1

We know 9 large hoses can fill 1 swimming pool in 4 hours.
Therefore, 9 large hoses can fill $\frac{1}{4}$ of the swimming pool in 1 hour.
Therefore, 1 large hose can fill $\frac{1}{9} \times \frac{1}{4} = \frac{1}{36}$ of the swimming pool in 1 hour.

We know 6 small hoses can fill 1 swimming pool in 8 hours.
Therefore, 6 small hoses can fill $\frac{1}{8}$ of the swimming pool in 1 hour.
Therefore, 1 small hose can fill $\frac{1}{6} \times \frac{1}{8} = \frac{1}{48}$ of the swimming pool in 1 hour.

Thus, the fraction of the pool that is filled in one hour using 4 large hoses and 8 small hoses is

$$4\left(\frac{1}{36}\right) + 8\left(\frac{1}{48}\right) = \frac{1}{9} + \frac{1}{6} = \frac{2}{18} + \frac{3}{18} = \frac{5}{18}$$

Since the pool is $\frac{5}{18}$ full in 1 hour, it will be completely full in $\frac{18}{5} = 3\frac{3}{5}$ hours or 3 hours and 36 minutes.

Solution 2

We know 9 large hoses can fill 1 swimming pool in 4 hours.
Therefore, 1 large hose can fill $\frac{1}{9}$ of the swimming pool in 4 hours.
Therefore, 4 large hoses can fill $\frac{1}{9}$ of the swimming pool in 1 hour.

We know 6 small hoses can fill 1 swimming pool in 8 hours.
Therefore, 1 small hose can fill $\frac{1}{6}$ of the swimming pool in 8 hours.
Therefore, 8 small hoses can fill $\frac{1}{6}$ of the swimming pool in 1 hour.

Thus, the fraction of the pool that is filled in one hour using 4 large hoses and 8 small hoses is

$$\frac{1}{9} + \frac{1}{6} = \frac{2}{18} + \frac{3}{18} = \frac{5}{18}$$

Since the pool is $\frac{5}{18}$ full in 1 hour, it will be completely full in $\frac{18}{5} = 3\frac{3}{5}$ hours or 3 hours and 36 minutes.