Problem of the Week
Problem A and Solution
International Jumping Relay

Problem

The International Jumping Relay Race will take place this year. In the race, each team has four members. The first member of each team starts the race and jumps until they tag the next member of their team. When a team member is tagged, they start jumping towards the next team member to tag them, except for the last team member who starts jumping towards the finish line.

BeHappy Bunny and Kicky Kangaroo are on different teams and will each be jumping in the last leg of the race. They are waiting 24 m away from the finish line for their teammates to tag them.

BeHappy Bunny is tagged first, and jumps three times before Kicky Kangaroo is tagged. Bunny's fourth jump and Kangaroo's first jump happen at the same time. BeHappy Bunny leaps 2 m forward with every jump, Kicky Kangaroo leaps 3 m forward with every jump, and the two animals take off and land at the same time on each jump.

(a) Which animal will get to the finish line first? Justify your answer.

(b) Will the two animals ever land at the same spot (not necessarily at the same time)? If so, how far away from the finish line are these spots?

Solution

One way to solve this problem is to make a table that keeps track of how far each animal has travelled after each jump. Recall that BeHappy Bunny jumps three times before Kicky Kangaroo jumps for the first time.

Bunny's Jump Number	Distance Jumped by Bunny (m)	Distance Jumped by Kangaroo (m)
1	2	0
2	4	0
3	6	0
4	8	3
5	10	6
6	12	9
7	14	12
8	16	15
9	18	18
10	20	21
11	22	24
12	24	27

(a) Based on the table, we see that Kicky Kangaroo will get to the finish line, which is 24 m away, on Bunny's 11th jump. Bunny will get to the finish line on their 12th jump. So Kicky Kangaroo will get to the finish line first.

Another way to determine the winner is to see how many jumps it takes for each to travel 24 m. We can do this with division:

BeHappy Bunny will take $24 \div 2 = 12$ jumps to get to the finish line.
Kicky Kangaroo will take $24 \div 3 = 8$ jumps to get to the finish line.

Since BeHappy Bunny jumps three times before Kicky Kangaroo starts, we can say that Bunny takes $12 - 3 = 9$ jumps to get to the finish line from the time that Kangaroo starts jumping. However, this is one more jump than it takes Kangaroo to get to the finish line. So Kicky Kangaroo will get to the finish line first.

(b) There are four spots where both animals will land. These are exactly 6 m, 12 m, 18 m, and 24 m from where the animals were waiting to be tagged. (Notice that these numbers are all common multiples of 2 and 3, which represent each animal's jumping distance.)

Therefore, the spots where they both land are $24 - 6 = 18$ m, $24 - 12 = 12$ m, $24 - 18 = 6$ m, and $24 - 24 = 0$ m from the finish line.