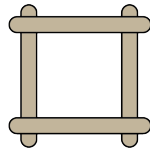# Problem of the Week
## Problem A and Solution
Crafty Construction

Problem

Sam is making square picture frames using popsicle sticks. He has one box of 50 popsicle sticks and wants to make 13 frames.

(a) Sam plans to make individual frames using four popsicle sticks as shown.

Will he have enough popsicle sticks to make 13 frames? Justify your answer.

(b) Sam changes his mind and instead of individual frames, he decides to connect the frames in a row. He starts by making a frame using four popsicle sticks, and then uses three popsicle sticks to create another frame attached to this frame. He then uses three more popsicle sticks to create another frame attached to the first two frames, so he has three connected frames, as shown.
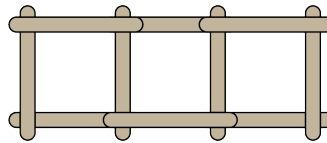
Sam plans to continue this process, using three more popsicle sticks for each frame, until he has 13 frames connected in a row. Will he have enough popsicle sticks to make 13 frames? Justify your answer.

(c) Can you draw another layout of the 13 frames that Sam could have built using at most 50 popsicle sticks? How many popsicle sticks does your layout use?

(d) Can you draw a layout of the 13 frames that uses fewer than 35 popsicle sticks?

Solution

(a) Since each frame requires 4 popsicle sticks, the total number of popsicle sticks required for 13 frames is equal to $13 \times 4 = 52$.

Alternatively, we could make a table showing the number of frames and the number of popsicle sticks required.

Number of Frames	Number of Popsicle Sticks
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32
9	36
10	40
11	44
12	48
13	52

Either way, since it will take 52 popsicle sticks to make 13 individual frames, Sam will not have enough popsicle sticks to make all the frames.

(b) The first frame uses 4 popsicle sticks but the other 12 frames use only 3 popsicle sticks each. So the total number of popsicle sticks required can be found by adding 4 to 12×3. Since $12 \times 3 = 36$, this gives a total of $4 + 36 = 40$ popsicle sticks.

Alternatively, we could make a table showing the number of frames and the number of popsicle sticks required.

Number of Frames	Number of Popsicle Sticks
1	4
2	7
3	10
4	13
5	16
6	19
7	22
8	25
9	28
10	31
11	34
12	37
13	40

Either way, since it will take 40 popsicle sticks to build a row of 13 connected frames, Sam will have enough popsicle sticks to make all the frames.
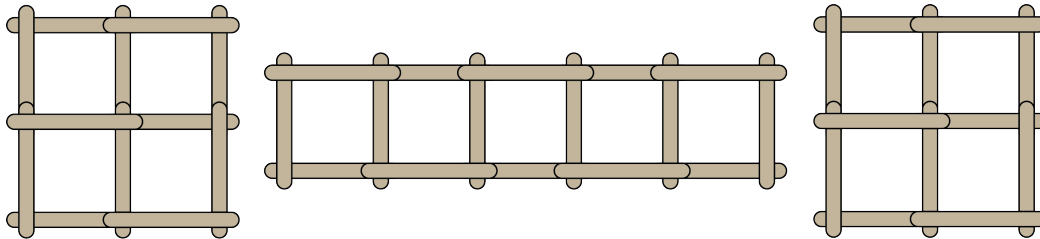
(c) There are many layouts of 13 frames you could make using at most 50 popsicle sticks. Two examples are shown.

The following layout uses 39 popsicle sticks.
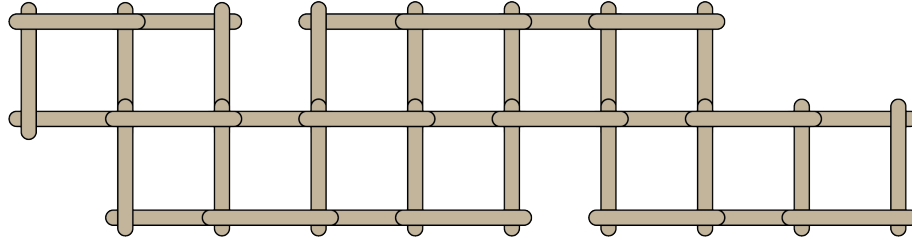
The following layout uses 40 popsicle sticks.

(d) The most efficient use of popsicle sticks is to share as many sides as possible with other frames. If you connect the frames to form a shape as close as possible to a square, you can make 13 frames using only 34 popsicle sticks, as shown.