Problem of the Week
Problem A and Solution
Patio Planning

Problem

Jody and Jillian are building a patio beside their house. They have square patio stones that have sides with a length of 50 cm. They have a total 75 patio stones available.

To build the patio, they place the patio stones so that two stones next to each other are touching and their sides are lined up exactly. They will only use whole patio stones and they want the entire patio covered in stones.

(a) If they want to have their patio in the shape of a square, what are the dimensions of the largest possible patio they can build?

(b) If they want to build a patio in the shape of a rectangle where the length of one side is twice as long as the length of the other side, what are the dimensions of the largest patio they can build?

Solution

(a) The following table shows the possible sizes of the patio if the patio is in the shape of a square.

Length	Width	Number of Stones Used
1	1	$1 \times 1 = 1$
2	2	$2 \times 2 = 4$
3	3	$3 \times 3 = 9$
4	4	$4 \times 4 = 16$
5	5	$5 \times 5 = 25$
6	6	$6 \times 6 = 36$
7	7	$7 \times 7 = 49$
8	8	$8 \times 8 = 64$
9	9	$9 \times 9 = 81$

Since there are only 75 patio stones available, the biggest square patio that Jody and Jillian can build is 8 patio stones long by 8 patio stones wide. Since each square patio stone has sides with length 50 cm, then 8 stones in a line have a length of $8 \times 50 = 400$ cm or 4 metres. So the largest square patio made out of 75 or fewer patio stones has dimensions 4 meters long by 4 metres wide.

(b) To investigate a patio that has one side that is twice as long as the other side, we can name the shorter side Length and the longer side Width. The following table shows the possible sizes of the patio.

Length	Width	Number of Stones Used
1	2	$1 \times 2 = 2$
2	4	$2 \times 4 = 8$
3	6	$3 \times 6 = 18$
4	8	$4 \times 8 = 32$
5	10	$5 \times 10 = 50$
6	12	$6 \times 12 = 72$
7	14	$7 \times 14 = 98$

Since we only have 75 patio stones, the biggest patio Jody and Jillian can build that has one side twice as long as the other side is a rectangle that is 6 patio stones long by 12 patio stones wide. Since each square patio stone has sides with length 50 cm, then 6 stones in a line have a length of $6 \times 50 = 300$ cm or 3 metres, and 12 stones in a line have a length of $12 \times 50 = 600$ cm or 6 metres. So the largest patio with one side twice as long as the other side and made out of 75 or fewer patio stones, has dimensions 3 metres by 6 metres.