



Problem of the Week

Problem C and Solution

More Flowers Please

Problem

A *perennial* is a plant that lives for multiple years. It grows back each spring from roots that go dormant over the autumn and winter. Leilani discovered two interesting species of perennials at the POTW Greenhouse called the Blue Starpoint and the Purple Parabola. After the Blue Starpoint goes dormant, it returns the following year as a Purple Parabola. After the Purple Parabola goes dormant, it returns the following year as two plants; one Blue Starpoint and one Purple Parabola. This cycle happens every year. Leilani planted two Blue Starpoints and three Purple Parabolas in her garden one spring. Assuming the plants behave exactly as described, and all of them continue to survive, how many Blue Starpoints and Purple Parabolas will be in her garden after 10 cycles?

Solution

Leilani started with 2 Blue Starpoints and 3 Purple Parabolas. In one year the 2 Blue Starpoints will become 2 Purple Parabolas. As well, the 3 Purple Parabolas will remain and produce 3 Blue Starpoints. So, after one cycle, there will be 3 Blue Starpoints and $2 + 3 = 5$ Purple Parabolas.

Proceeding from year one to year two, the 3 Blue Starpoints will become 3 Purple Parabolas. As well, the 5 Purple Parabolas will remain and produce 5 Blue Starpoints. So, after two cycles, there will be 5 Blue Starpoints and $3 + 5 = 8$ Purple Parabolas.

At this point we can make an observation. The number of Blue Starpoints in a given year equals the number of Purple Parabolas in the previous year. Also, the number of Purple Parabolas in a given year equals the sum of the Blue Starpoints and Purple Parabolas in the previous year. We can use this observation to make a table for the remaining years.

Year Number	Number of Blue Starpoints	Number of Purple Parabolas
0	2	3
1	3	5
2	5	8
3	8	13
4	13	21
5	21	34
6	34	55
7	55	89
8	89	144
9	144	233
10	233	377

After ten cycles, there will be 233 Blue Starpoints and 377 Purple Parabolas, for a total of $233 + 377 = 610$ plants in Leilani's garden. Hopefully she has a big garden!



EXTENSION:

This problem was inspired by a past [Beaver Computing Challenge \(BCC\)](#) problem.

The number of a particular flower type in a specific year is dependent on the number of flowers of each of the types from the previous year. This is an example of a *recursion*.

A famous example of a recursion is known as the *Fibonacci Sequence*. The first two numbers (or *terms*) in the sequence of numbers are defined. They are both 1. Each remaining term in the sequence is equal to the sum of the two previous terms.

So, the third term is equal to the sum of the first and second terms, and is therefore $1 + 1 = 2$.

The fourth term is equal to the sum of the second and third terms, and is therefore $1 + 2 = 3$.

The fifth term is equal to the sum of the third and fourth terms, and is therefore $2 + 3 = 5$.

We can continue generating more terms in the sequence in this manner.

The first 15 Fibonacci numbers are

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, and 610.

In our problem, the number of Purple Parabolas in a given year equals the sum of the number of Blue Starpoints and Purple Parabolas in the previous year. If we had started with only 1 Blue Starpoint and 0 Purple Parabolas, the number of Purple Parabolas after each cycle would match the terms in the Fibonacci sequence. Try it out!