Problem of the Week

Problem C and Solution

Everything in its Place 1

Problem

(a) A Venn diagram has two circles, labelled A and B. Each circle contains integers that satisfy the following criteria.

A: Less than $-\frac{7}{6}$

B: Greater than $-\frac{1}{4}$

The overlapping region in the middle contains integers that are in both A and B, and the region outside both circles contains integers that are neither in A nor B. In total this Venn diagram has four regions. Place integers in as many of the regions as you can. Is it possible to find an integer for each region?

(b) A Venn diagram has three circles, labelled A, B, and C. Each circle contains pairs of integers that satisfy the following criteria.

A: Their sum is negative

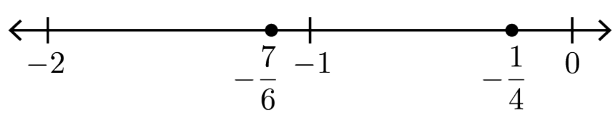
B: Their product is negative

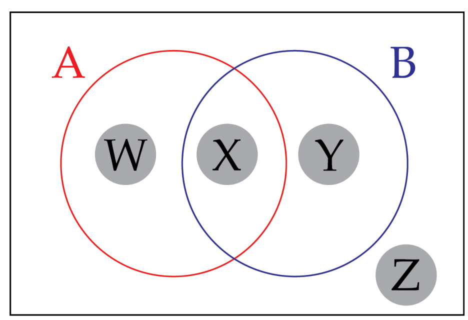
C: Their difference is even

In total this Venn diagram has eight regions. Place pairs of integers in as many of the regions as you can. Is it possible to find a pair of integers for each region?

Solution

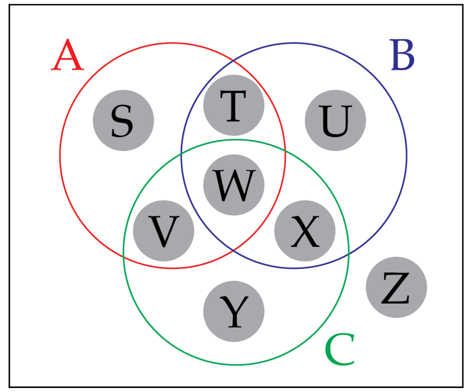
(a) We have marked the four regions W, X, Y, and Z.
We plot the given fractions on a number line as a reference:





- Any integer in region W must be less than $-\frac{7}{6}$ and *not* greater than $-\frac{1}{4}$. This means the integer must be less than $-\frac{7}{6}$ and less than or equal to $-\frac{1}{4}$. Any integer less than or equal to -2 will satisfy this. Some examples are $-2, -3$, and -10.
- Any integer in region X must be less than $-\frac{7}{6}$ and greater than $-\frac{1}{4}$. It is not possible to find such an integer so this region must remain empty.
- Any integer in region Y must be greater than $-\frac{1}{4}$ and *not* less than $-\frac{7}{6}$. This means the integer must be greater than $-\frac{1}{4}$ and greater than or equal to $-\frac{7}{6}$. Any integer greater than or equal to 0 will satisfy this. Some examples are $0, 1$, and 30.
- Any integer in region Z must be *not* less than $-\frac{7}{6}$ and *not* greater than $-\frac{1}{4}$. This means the integer must be greater than or equal to $-\frac{7}{6}$ and less than or equal to $-\frac{1}{4}$. The only integer that satisfies this is -1.

(b) We have marked the eight regions S, T, U, V, W, X, Y, and Z. It is helpful if we first think about the pairs of integers that each circle could contain. Two integers have a negative sum if they are both negative, or they have different signs and the negative number is larger in magnitude than the positive number. Two integers have a negative product if they have different signs. Two integers have an even difference if they are both even or both odd, regardless of their signs, and regardless of which number is being subtracted from the other.



- Any pair of integers in region S must have a negative sum, a positive product, and an odd difference. This means they must both be negative, and one must be even and the other must be odd. One example is -5 and -6, because $(-5)+(-6) = -11 < 0$, $(-5)\times(-6) = 30 > 0$, and $(-5)-(-6) = 1$, which is odd.
- Any pair of integers in region T must have a negative sum, a negative product, and an odd difference. This means they must have different signs, and the negative number must be larger in magnitude than the positive number. Also one number must be even and the other must be odd. One example is 3 and -8, because $3+(-8) = -5 < 0$, $3\times(-8) = -24 < 0$, and $3-(-8) = 11$, which is odd.
- Any pair of integers in region U must have a positive sum, a negative product, and an odd difference. This means they must have different signs, and the positive number must be larger in magnitude than the negative number. Also one number must be even and the other must be odd. One example is 8 and -3, because $8+(-3) = 5 > 0$, $8\times(-3) = -24 < 0$, and $8-(-3) = 11$, which is odd.
- Any pair of integers in region V must have a negative sum, a positive product, and an even difference. This means they must both be negative, and they must be either both even or both odd. One example is -4 and -6, because $(-4)+(-6) = -10 < 0$, $(-4)\times(-6) = 24 > 0$, and $(-4)-(-6) = 2$, which is even.
- Any pair of integers in region W must have a negative sum, a negative product, and an even difference. This means they must have different signs, and the negative number must be larger in magnitude than the positive number. Also they must be either both even or both odd. One example is 2 and -8, because $2+(-8) = -6 < 0$, $2\times(-8) = -16 < 0$, and $2-(-8) = 10$, which is even.
- Any pair of integers in region X must have a positive sum, a negative product, and an even difference. This means they must have different signs, and the positive number must be larger in magnitude than the negative number. Also they must be either both even or both odd. One example is 8 and -2, because $8+(-2) = 6 > 0$, $8\times(-2) = -16 < 0$, and $8-(-2) = 10$, which is even.
- Any pair of integers in region Y must have a positive sum, a positive product, and an even difference. This means they must both be positive, and either both even or both odd. One example is 5 and 3, because $5+3 = 8 > 0$, $5\times 3 = 15 > 0$, and $5-3 = 2$, which is even.
- Any pair of integers in region Z must have a positive sum, a positive product, and an odd difference. This means they must both be positive, and one must be even and the other must be odd. One example is 5 and 4, because $5+4 = 9 > 0$, $5\times 4 = 20 > 0$, and $5-4 = 1$, which is odd.