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Problem of the Week
Problem C and Solution
Hundred Deck 1

Problem

Hundred Deck is a deck consisting of 100 cards numbered from 1 to 100. Each card has the same number printed on both sides. One side of the card is red and the other side of the card is yellow.

Sarai places all of the cards on a table with each card's red side facing up. She first flips over every card that has a number on it which is a multiple of 2. She then flips over every card that has a number on it which is a multiple of 3.

After Sarai has finished, how many cards have their red side facing up?

Solution

After flipping over all of the cards with numbers that are multiples of 2, 50 cards have their red side facing up and 50 cards have their yellow side facing up. All of the cards with their red side facing up are numbered with an odd number. All of the cards with their yellow side facing up are numbered with an even number.

Next, in the second round of flips, Sarai flips over every card that is numbered with a multiple of 3. Let's look at how many cards with their red side facing up will be flipped over to yellow and how many cards with their yellow side facing up will be flipped over to red.

There are 33 multiples of 3 from 1 to 100. They are

$$3, 6, 9, 12, 15, \ldots, 87, 90, 93, 96, 99$$

Of these numbers, 17 are odd and 16 are even. The 17 odd multiples of 3 currently have their red side facing up, and therefore are flipped over to yellow. The 16 even multiples of 3 currently have their yellow side facing up, and are therefore flipped over to red (again).

So, after the first flip there were 50 cards with their red side facing up and 50 cards with their yellow side facing up. Of the 50 red, 17 were flipped to yellow. Of the 50 yellow, 16 were flipped to red. Therefore, after Sarai has finished, $50 - 17 + 16 = 49$ cards have their red side facing up.