



Problem of the Week

Problem C and Solution

Balloons

Problem

Liang has five balloons that are identical, except for their colour. Three are red (each labelled with an R) and two are green (each labelled with a G). He wants to put the five balloons in a row, but he is not sure which order he likes the best. How many different ways are there to arrange the five balloons in a row?

Solution

We will consider the following cases:

1. If the first balloon is green, then there are four positions where the second green balloon could go. Once the green balloons are placed, the remaining three balloons must be red. Therefore, there are 4 ways to arrange the balloons so that the first balloon is green.
2. If the first balloon is red and the second balloon is green, then there are three positions where the second green balloon could go. Once the green balloons are placed, the remaining balloons must be red. Therefore, there are 3 ways to arrange the balloons so that the first balloon is red and the second balloon is green.
3. If the first two balloons are red and the third balloon is green, then there are two positions where second green balloon could go. Once the green balloons are placed, the remaining balloon must be a red balloon. Therefore, there are 2 ways to arrange the balloons so that the first two balloons are red and the third balloon is green.
4. If the first three balloons are red and the fourth balloon is green, then the fifth balloon must be the second green balloon. Therefore, there is only 1 way to arrange the balloons so that the first three balloons are red and the fourth balloon is green.

There are no other cases to consider. The total number of ways to arrange the balloons is the sum of the number of ways from each of the cases. Therefore, there are $4 + 3 + 2 + 1 = 10$ ways to arrange the balloons in a row.