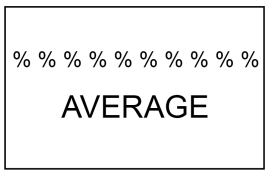

Problem of the Week
Problem D and Solution
Another Average

Problem

The numbers 2124, 1984, 1742, 2344, 2074, and 1632 are each written on a card. Daniyal takes four of the cards and calculates the mean (average) of their numbers to be 2021. Determine the mean of the numbers on the remaining two cards.

Extra Problem: Can you interpret the picture puzzle above?

Solution

At the outset, it should be noted that we could "play" with the numbers to determine which of the four numbers have an average of 2021. We could then easily determine the average of the remaining two numbers. This method works decently well on a problem with a small number of numbers. However, if we were to increase the size of the list by just a few more numbers, then the task would not be easily solved using this approach. It turns out, we can solve this problem without actually figuring out which four numbers Daniyal used.

The sum of all six numbers is

$$2124 + 1984 + 1742 + 2344 + 2074 + 1632 = 11\,900$$

Since the average of four of the numbers is 2021, then the sum of those four numbers is $4 \times 2021 = 8084$.

The sum of the two remaining numbers is $11\,900 - 8084 = 3816$. Since there are two numbers in the sum, the average of the two numbers is calculated by dividing the sum by 2. The average of the remaining two numbers is then $3816 \div 2 = 1908$.

Although not required, the two numbers that sum to 3816 are 1742 and 2074. It is then easily verified that the average of the four other numbers, 2124, 1984, 2344, and 1632, is 2021.

Extra Problem Answer: Ten percent above average.