Problem of the Week
Problem A and Solution
A Hop, Skip, and a Jump!

Problem

Alexis and her friends Mikai, Sophia, and Casper enter a team competition that involves hopping, skipping, biking, and rollerblading for a total of 3 km. Each team member picks an activity and must take turns completing a section of the course by doing their activity.

Alexis will hop for 100 m, then Mikai will bike the next 150 m, followed by Sophia who will rollerblade for the next 200 m, and finally Casper will skip the next 50 m. They will repeat their activity until the team completes the 3 km race.

(a) How many times does each member have to do their activity to complete the race?

(b) What fraction of the race does each team member complete? Put the fractions in order from least to greatest.

Solution

(a) The total distance travelled after each participant has completed the activity once is $100 + 150 + 200 + 50 = 500$ m. We know that 3 km is equal to $3 \times 1000 = 3000$ m. We can use skip counting by 500 to see how many times they have to complete the cycle of four activities to travel 3000 m: 500, 1000, 1500, 2000, 2500, 3000. This means that each team member would need to do their activity 6 times to complete the race.

(b) One way to determine the fractional amounts is to focus on the distance of one cycle of the race, which is 500 m. Then we can take each distance the individuals travelled as the numerator of the fraction and 500 as the denominator of the fraction.

The fractions are: hopping $\frac{100}{500}$, biking $\frac{150}{500}$, rollerblading $\frac{200}{500}$, and skipping $\frac{50}{500}$.

Alternatively, we can determine the fractional amounts using the entire 3000 m. Then we would have found that the fractions are: hopping $\frac{600}{3000}$, biking $\frac{900}{3000}$, rollerblading $\frac{1200}{3000}$, and skipping $\frac{300}{3000}$.

We could also reduce each of these fractions by dividing the numerator and denominator by the same number to get the following fractions: hopping: $\frac{600}{3000} = \frac{100}{500} = \frac{1}{5}$, biking: $\frac{900}{3000} = \frac{150}{500} = \frac{3}{10}$, rollerblading: $\frac{1200}{3000} = \frac{200}{500} = \frac{2}{5}$, and skipping: $\frac{300}{3000} = \frac{50}{500} = \frac{1}{10}$.

We can compare the fractions in many different ways to determine the relative sizes. It is easy to compare fractions with the same denominator, which we have with the fractions with 500 or 3000 as the denominators.

Arranging these in order from least to greatest we get:

skipping	hopping	biking	rollerblading
$\frac{300}{3000} = \frac{50}{500} = \frac{1}{10}$,	$\frac{600}{3000} = \frac{100}{500} = \frac{1}{5}$,	$\frac{900}{3000} = \frac{150}{500} = \frac{3}{10}$,	$\frac{1200}{3000} = \frac{200}{500} = \frac{2}{5}$.