Problem of the Week

Problem A and Solution

Tracking Triangles

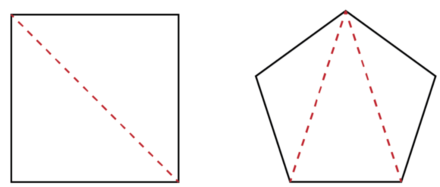
Problem

A regular polygon is a closed shape where all the side lengths are the same. Pauline draws lines inside regular polygons according to the following rules.

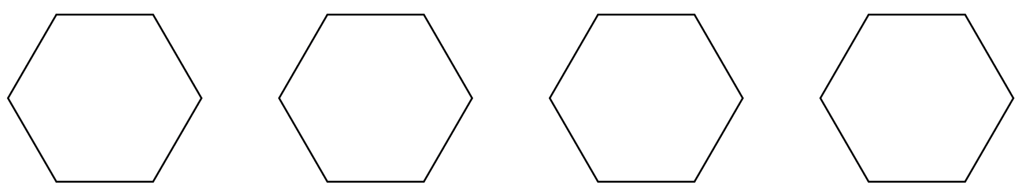
1. The lines must connect two vertices that are *not* beside each other.

2. The lines must be straight and cannot cross.

Pauline continues to draw lines until she cannot draw any more. At this point, the inside of her polygon will be made up entirely of triangles. For example, after drawing lines in a square she creates 2 triangles, and after drawing lines in a regular pentagon she creates 3 triangles, as shown.



(a) Notice that if Pauline had drawn lines between different pairs of vertices in the square and the regular pentagon, the resulting diagrams would have been rotations or reflections of the diagrams above, but would otherwise have been the same. Is it possible for Pauline to draw lines in a regular hexagon and create more than one diagram, which cannot be obtained from the others by a rotation or reflection? Use the hexagons below to test it out.



(b) Look at the number of triangles Pauline creates in a square, pentagon, and hexagon. Use this to predict the number of triangles Pauline creates in an octagon, and then check to see if you are correct.

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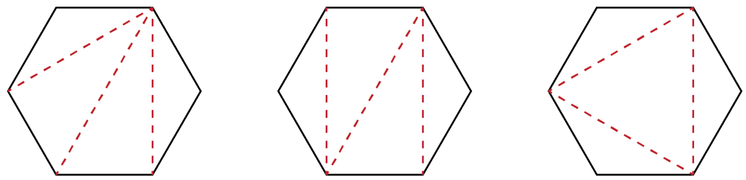
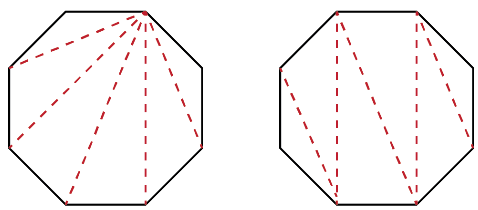
Solution

(a) There are three different diagrams that can be created by drawing lines in a regular hexagon. They are shown below.



All other possible diagrams are reflections or rotations of one of these three.

(b) Pauline creates 2 triangles in a square, 3 triangles in a pentagon, and 4 triangles in a hexagon. It appears as though the number of triangles is always 2 less than the number of sides in the polygon. Then we can predict that Pauline can create $8 - 2 = 6$ triangles in an octagon. In fact this is true. Two examples are shown.



Teacher's Notes

It takes some advanced mathematics to actually prove that in any regular polygon the **maximum** number of triangles we can draw according to the rules is always 2 less than the number of sides of the polygon. However, we can informally convince ourselves that it is possible to draw **at least** this many triangles. In general, one way we can draw the triangles inside the polygon is to pick one vertex as the end point of all the lines and draw a line to each of the other vertices in the polygon that are not adjacent to that vertex. If we examine that pattern, we see we always end up with the number of triangles being 2 less than the number of sides in the polygon.

Remembering this pattern can help us determine another feature of a polygon. It is known that the sum of the interior angles a triangle is always $180°$. You can test this by drawing many different triangles and measuring the interior angles within them. This can also be proven true for all triangles without having to measure specific angles, but again we need some more rules of geometry to do so. However, knowing this we can figure out the sum of the interior angles of any polygon by using the pattern from this problem. For example, since we can draw 4 triangles inside a hexagon, then the sum of the interior angles of a hexagon must be $4 \times 180° = 720°$. Similarly, the sum of the interior angles of a *icosagon* (a 20-sided polygon) is $18 \times 180° = 3240°$. In general, the sum of the interior angles of a polygon with n sides is $(n - 2) \times 180°$.