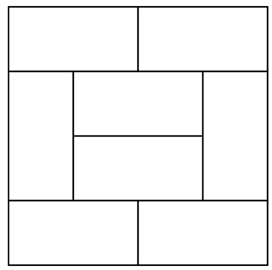
Problem of the Week
Problem B and Solution
Mystery Dimensions

Problem

Eight congruent rectangles are arranged to form a larger rectangle as shown.



(a) If the congruent rectangles each have a length of 6 cm and a width of 3 cm, what is the perimeter of the larger rectangle?

(b) Suppose that the congruent rectangles each have a longer side of length L cm and a shorter side of length 4 cm. Suppose also that the perimeter of the larger rectangle is 64 cm.

(i) What is the value of L?

(ii) What is the area of one of the eight congruent rectangles?

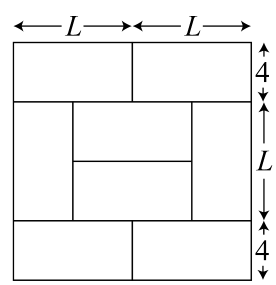
Extension: Can you solve part (b) without knowing that the length of the shorter side of each rectangle is 4 cm? If so, how?

Solution

(a) Since each rectangle has a length of 6 cm and a width of 3 cm, the larger rectangle must have sides of lengths $6 + 6 = 12$ cm and $3 + 6 + 3 = 12$ cm. Thus, the perimeter of the larger rectangle is $12 + 12 + 12 + 12 = 48$ cm.

(b)

(i) Since each rectangle has a longer side of length L cm and shorter side of length 4 cm, we can label our diagram to find the dimensions of the larger rectangle. Using this, we determine that the lengths of the sides of the larger rectangle are $L + L = 2L$ and $4 + L + 4 = L + 8$. Since we know the perimeter of the larger rectangle is 64 cm, we can write the following equation.



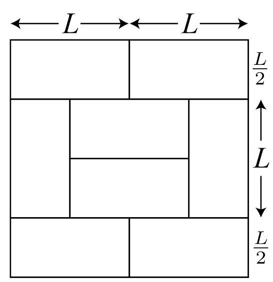
$$\begin{aligned} 2L + 2L + (L+8) + (L+8) &= 64 \\ 6L + 16 &= 64 \\ 6L &= 64 - 16 \\ 6L &= 48 \end{aligned}$$

Since $6 \times 8 = 48$, it follows that $L = 8$ cm.

(ii) The area of a rectangle is equal to its length times its width. Thus, the area of each congruent rectangle is $8 \times 4 = 32$ cm^2.

Extension Solution:

If we ignore the two rectangles on the top and the two rectangles on the bottom, we can see that two rectangles placed on top of each other horizontally have a height of L. Therefore, the shorter side of each rectangle equals half its longer side, or $\frac{L}{2}$. We can label our diagram to find the dimensions of the larger rectangle.



Using this, we determine that the larger rectangle has sides of length $L + L = 2L$ and $\frac{L}{2} + L + \frac{L}{2} = 2L$. So the larger rectangle is actually a square with side length $2L$. Since we know its perimeter is 64 cm, it follows that $2L + 2L + 2L + 2L = 64$, or $8L = 64$. Since $8 \times 8 = 64$, it follows that $L = 8$ cm. So, we can solve this problem without knowing the width of each rectangle.