



Problem of the Week

Problem B and Solution

A Pocketful of Coins

Problem

Dakarai has a some Canadian coins in his pocket: one nickel (worth \$0.05), one dime (worth \$0.10), one quarter (worth \$0.25), one loonie (worth \$1.00), and one toonie (worth \$2.00).



- (a) Suppose he reaches into his pocket and pulls out one coin at random.
- What is the probability that he will pull out
 - a nickel?
 - a quarter?
 - a toonie?
 - What is the probability that the total value of the coins remaining in his pocket is
 - less than \$1.00?
 - greater than \$1.35?
 - less than \$2.00?
- (b) Suppose Dakarai reaches into his pocket and pulls out two coins at random. Which is greater, the probability that the coins in his hand have a value of \$0.35, or the probability that the coins in his hand have a value of \$3.00?



Solution

(a) Dakarai is selecting one of the five coins ‘at random’.

(i) Since his selection is ‘at random’, there is an equal chance he will pull out any one of the coins, so the probability for each of these is equal to $\frac{1}{5} = 0.2$, or 20%.

- (ii)
- There is no combination of any four of the coins that has a total value less than \$1.00. Therefore, this probability is equal to 0.
 - If Dakarai draws the coin of greatest value (the toonie), the total value of the remaining coins will be $\$(1.00 + 0.25 + 0.10 + 0.05) = \1.40 , which is greater than \$1.35. So the total value of the remaining coins will always be greater than \$1.35, regardless of which coin he chooses. Therefore, this probability is equal to 1, or 100%.
 - If Dakarai picks only one coin, the only way the remaining coins could have total value less than \$2.00 is if he pulls out the toonie. Thus, the probability is $\frac{1}{5} = 0.2$, or 20%.

(b) There are exactly two coins with total value \$0.35, namely the dime and the quarter. Similarly, there are exactly two coins with total value \$3.00, namely the loonie and the toonie. Since the coins are drawn ‘at random’, the probabilities of these events must be equal.

NOTE: The actual probability of each event is 0.1 or 10%. This can be illustrated by constructing a tree diagram, or by the following argument. The probability of drawing the dime first is $\frac{1}{5}$. Then there are only four coins in his pocket, so the probability of drawing the quarter next is $\frac{1}{4}$. Thus, the probability of drawing the dime and then the quarter is $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20} = 0.05$, or 5%. Similarly, the probability of drawing the quarter and then the dime is $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20} = 0.05$, or 5%. Thus, the total probability of drawing the dime and quarter is $0.05 + 0.05 = 0.1$, or 10%. A similar analysis can be used to show that the total probability of drawing the loonie and toonie is also 0.1, or 10%.